Graded. +2

INTERMEDIATE ALGEBRA

GPS #1 **1.1 Describing Data with Set of Numbers** **NAME:** Kelly Fenton

Properties of Real Numbers:

* Identity (0): The identity for addition is 0; $a+0=a$
* Identity (1): The identity for multiplication is 1; $a\cdot 1=a$
* Commutative: $a+b=b+a$ and $a\cdot b=b\cdot a$
* Associative: $(a+b)+c=a+(b+c)$ and $(a\cdot b)\cdot c=a\cdot(b\cdot c)$
* Distributive: $a(b+c)=ab+ac$ and $a(b-c)=ab-ac$

Note: a,b,c are real numbers.

20/20 Good job!

1. Complete the sets of numbers for the following:

a) Natural Numbers $=\{1,2,3,4,\ldots\}$ Counting numbers

b) Whole Numbers $=\{0,1,2,3\ldots\}$ just add zero

c) Integers $=\{\ldots,\ ,\ ,\ ,1,\ ,\ ,\ldots\}$ include negatives

2. Classify each real number as one or more of the following: natural number, whole number, integer, rational number or irrational number.

$$-1,\frac{3}{2},0,\frac{7}{13},2,-10,-\sqrt{3},-\pi,8,\sqrt{9}$$

Natural numbers: 2, 8, $\sqrt{9}=3$ (no decimals, fractions, negatives, etc.)

Whole numbers: 0, 2, 8, $\sqrt{9}=3$ (regular #'s w/out ~~0 or~~ Neg. but with 0.)

Integers: ~~Include negative numbers~~ 0, -1, -10, 2, 8, $\sqrt{9}=3$

Rational numbers: $-1,\frac{3}{2},0,\frac{7}{13},2,-10,8,\sqrt{9}=3$ (all #'s but irrational ones)

Can be put into a fraction

Irrational numbers: $-\sqrt{3}$, $-\pi$ (real #'s that aren't rational $\sqrt{15}$, 1.1711711171.....)

3. State the property of real numbers that justifies the following statements.

a) $(2\cdot 4)\cdot x=2\cdot(4\cdot x)$ Associative

b) $3+p=p+3$ Commutative

c) $(1\cdot 3)\cdot 6=3\cdot 6$ Identity of 1

d) $(3+2)+9=3+(2+9)$ Associative

4. Apply the distributive property to the following:

a) $2(x+5)=$ $2x+10$

b) $7x-3x+x=$ $x(7-3+1)$ $5x$

c) $8-3(x+3)=$ $8-3x-9$ $-3x-1$

d) $5-2(x-2)=$ $5-2x+4$ $9-2x$ ✓

5. Calculate the average of the list of numbers.

a) $2,5,11,6$ $\frac{24}{4}=6$

b) $14,15,17,18,26$ $=\frac{90}{5}=18$

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GPS #2 **1.2 Operations on Real Numbers** **NAME:** Kelly Fenton

Some Useful Guidelines:

* Absolute Value $|a| = \begin{cases} a, a \geq 0 \\ -a, a < 0 \end{cases}$

* Additive Inverse: The additive inverse (or opposite) of a real number a is $-a$.

* Multiplicative Inverse: The multiplicative inverse (or reciprocal) of a real number a is $\frac{1}{a}$.

* Division: $\frac{a}{b} = a \cdot \frac{1}{b}$

Note: a, b, c are real numbers.

20/20

Evaluate the following:

1. a) $|-8|+3 = 11$ b) $|-3|+|-2|+1 = 6$ c) $|5|+|-7|+2 = 14$

2 a) $|6-8+1| = 1$ b) $|4.5-5+1| = .5$ c) $|-2+3.2-5| =$ 3.8

3. Find the additive inverse, or opposite, of the following:

a) $347 = -347$ b) $-\frac{5}{6} = \frac{5}{6}$ c) $2-3x = -2+3x$ c) $-a+b = a-b$

4. Find the multiplicative inverse, or reciprocal, of the following:

a) $\frac{8}{9} = \frac{9}{8}$ b) $-\frac{3}{4} = -\frac{4}{3}$ c) $\frac{3}{x-1} = \frac{x-1}{3}$ d) $4a = \frac{1}{4a}$

5. Evaluate the following arithmetic operations and simplify.

a) $-2+1+(-3)+6 = 2$

b) $\frac{-3}{2} - \left(\frac{-1}{2}\right) = \frac{-2}{2} = -1$

c) $-9 \div -3 = \frac{9}{3} = \frac{9}{1} \cdot \frac{1}{3}$

$= 3$

d) $\left(\frac{4}{3} \cdot \frac{3}{8}\right) \div \frac{1}{2} = \frac{1}{2} \div \frac{1}{2}$

$= \frac{1}{2} \cdot \frac{2}{1} = \frac{2}{2} = 1$

e) $7x-2x+x$

$= x(7-2+1)$

$= 6x$

f) $\frac{1}{2}x - \frac{1}{4}x = \frac{1}{4}x$ or $\frac{x}{4}$

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GPS #3 **1.3 INTEGER EXPONENTS** **NAME:** Kelly Fenton

Useful Definitions and Rules: For all integers m and n and all real numbers a and b:

* Exponential Expression: $2^4 = 2\cdot2\cdot2\cdot2$ (4 factors of 2), $3^5 = 3\cdot3\cdot3\cdot3\cdot3$ (5 factors of 3), $a^5 = a\cdot a\cdot a\cdot a\cdot a$ (5 factors of a), $a^n = a\cdot a\cdot a\cdot a\cdot a...a$ (n factors of a)
* Product Rule: $a^m \cdot a^n = a^{m+n}$ ← Always add exponents during mult.
* Zero Exponent: $a^0 = 1 (a \neq 0)$
* Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$
* Negative Exponent: $a^{-n} = \frac{1}{a^n} (a \neq 0)$
* Power Rules: $(a^m)^n = a^{mn}$; $(ab)^m = a^m b^m$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} (b \neq 0)$$

A number is in scientific notation when it is written as a product of a number between 1 and 10 (inclusive of 1) and an integer power of 10. For example: 2.35×10^{10}

Good job!

Assume all variables represent nonzero real numbers for the following:

1. Evaluate the following:

a) $2^3 = 8$ b) $(-3)^4 = 81$ c) $-4^3 = -64$

d) $\sqrt{25} = 5 = (5^2)^{\frac{1}{2}}$ e) $\sqrt[3]{8} = (2^3)^{\frac{1}{3}} = 2$ f) $\sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{(4^2)^{\frac{1}{2}}}{(3^2)^{\frac{1}{2}}} = \frac{4}{3}$

2. Apply the product rule for exponents, if possible, in each case.

a) $4^5 \cdot 4^2 = 4^7$ b) $7^3 \cdot 7^9 \cdot 7^2 = 7^{14}$

c) $(-4x^3)(6x^2) = -24x^5$ d) $(2x^5)(y^2) = 2x^5y^2$

3. Apply the quotient rule for exponents and write each result using only positive exponents.

a) $\frac{2^5}{2^3} = 2^2 = 4$ b) $\frac{7^{-2}}{7^8} = \frac{1}{7^{10}}$

c) $\frac{x^8}{x^3} = x^5$ d) $\frac{y^4}{y^{-2}} = y^6$

3. Use one or more power rules to simplify each expression.

a) $(y^5)^2 = y^{10}$ b) $(x^{-2})^3 = x^{-6} = \frac{1}{x^6}$

c) $(3x)^4 = 81x^4$ d) $\left(\frac{6}{-5}\right)^2 = \frac{6^2}{(-5)^2} = \frac{36}{25}$

4. Evaluate and simplify each expression so that no negative exponents appear in the final result

a) $-(-7)^0 = -1$ b) $2x^0 - y^0 = 1$

c) $(2k^{-1})^4 = \left(\frac{2}{k^1}\right)^4 = \frac{16}{k^4}$ d) $\left(\frac{6x^{-2}}{x^{-3}}\right)^2 = \left(\frac{6x^3}{x^2}\right)^2 = (6x)^2 = 36x^2$ Subtract same base

5. Write each number in scientific notation.

a) $3600 = 3.6\times10^3$ b) $-790{,}000 = -7.9\times10^5$

c) $0.000896 = 8.96\times10^{-4}$ d) $-0.0000555 = -5.55\times10^{-5}$

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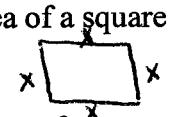
GPS #4 **1.4 VARIABLE, EQUATIONS, AND FORMULAS** **NAME:** Kelly Fenton

Useful Terminologies:

* Variable: Represents an unknown quantity. [Example: x, y, z, A, B, C.]
* Algebraic Expression: Consists of numbers, variables, operation symbols, and grouping symbols. [Example: $3x - 5$]
* Equation: A statement that two algebraic expressions are equal – contains an equal sign. [Example: $3x - 5 = y$]
* Formula: An equation used to calculate one quantity, using known values of other quantities. [Example: $P = 2L + 2W; A = \pi r^2; C = 2\pi r$] Square = b·h

1. Write a formula for the following:

a) Find the area of a square with a side of x.



$= X \cdot X = X^2$

$A = x^2$

b) Find the area of a circle with a radius a.

$= \pi a^2$

c) Find the circumference of a circle with a radius b.

$= 2\pi b$

2. Evaluate the formula for the given value(s) of the variable(s).

a) $P = 2L + 2W; L = 3, W = 4$

$P = 2(3) + 2(4)$

$P = 6 + 8$

$P = 14$

b) $y = 2x + 1; x = 8$

$y = 2(8) + 1$

$y = 16 + 1$

$y = 17$

c) $A = \frac{1}{2}bh; b = 6, h = 2$

$A = \frac{1}{2}(6)(2) \quad A = \frac{1}{2}(12) = 6$

d) $y = x^2; x = 5$

$y = 5^2$

$y = 25$

3. a) Find a value for a so that $y = ax$ models the data.

x	-2	-1	0	1	2
y	-8	-4	0	4	8

$y = ax$

$\frac{-8}{-2} = \frac{a(-2)}{-2} \quad a = 4$

$\frac{-4}{-1} = \frac{a(-1)}{-1} \quad a = 4$

$y = 4x$

b) Find a value for a so that $y = ax$ models the data.

x	2	3	4	5	6
y	-4	-6	-8	-10	-12

$\frac{(-4)}{2} = \frac{a(2)}{2} \quad -2 = a$

$\frac{-6}{3} = \frac{a(3)}{3} \quad -2 = a$

$y = -2x$

4. a) Write an equation that models the data.

$y = ax$

x	1	2	3	4	5
y	2	4	6	8	10

$\frac{2}{1} = \frac{a(1)}{1} \quad 2 = a$

$\frac{4}{2} = \frac{a(2)}{2} \quad 2 = a$

$y = 2x$

b) Write an equation that models the data.

$y = ax$

x	-2	-1	0	1	2
y	4	1	0	1	4

$\frac{4}{-2} = \frac{a(-2)}{-2} \quad -2$

$\frac{1}{-1} = \frac{a(-1)}{-1} \quad -1 = a$

$\frac{4}{2} = \frac{a(2)}{2} \quad 2 = a$

$y = ax$

$a = x$

$y = x \cdot x$

$y = x^2$

INTERMEDIATE ALGEBRA

GPS #5 **1.5 INTRODUCTION TO GRAPHING** **NAME:** Kelly Fenton

Useful Terminologies:

* Relation: A set of ordered pairs. [Example: $S = \{(-1,3),(2,4),(3,-2),(4,5)\}$]
* Domain: In a relation consisting of ordered pairs (x,y), the set of x-values is the domain.
* Range: In a relation consisting of ordered pairs (x,y), the set of y-values is the range.

x=domain
y=range

Domain Range S R D R D R

18

1. Identify the domain and range of the following:

a) $S_1 = \{(0,4),(3,9),(6,-2),(4,5)\}$

Set

D={0, 3, 6, 4} } members of a set

R={4, 9, -2, 5} }

b) $S_2 = \{(-1,2),(1,3),(5,-1),(9,2)\}$

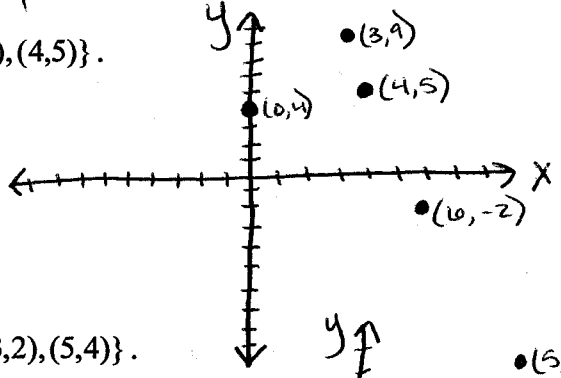
Set

D={-1, 1, 5, 9}

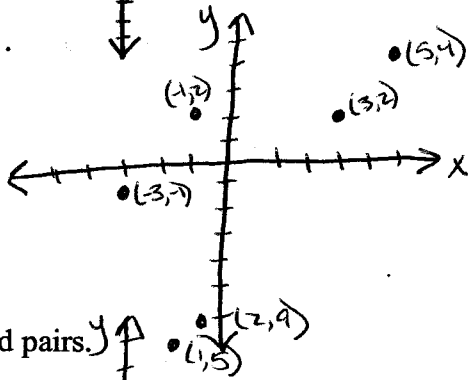
R={2, 3, -1}

= only 1 (must be unique members)

2. a) Make a scatterplot of the relation $S = \{(0,4),(3,9),(6,-2),(4,5)\}$.

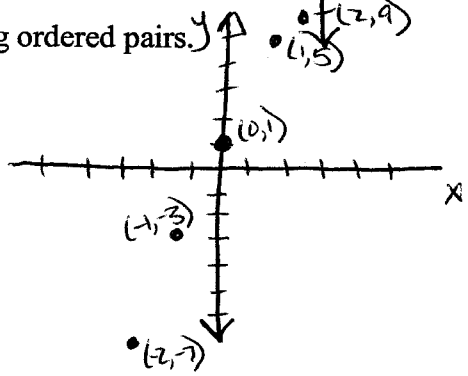


b) Make a scatterplot of the relation $S = \{(-3,-1),(-1,2),(3,2),(5,4)\}$.



3. Evaluate $y = 4x+1$ for $x = -2,-1,0,1$ and 2. Plot the resulting ordered pairs.

x	y	(x,y)
-2	-7	(-2,-7)
-1	-3	(-1,-3)
0	1	(0,1)
1	5	(1,5)
2	9	(2,9)



4. The formula $C = \frac{5}{9}(F-32)$ can be used to convert degrees Fahrenheit, F, to degrees Celsius, C. If the outside temperature is $14^\circ F$, find the equivalent temperature in Celsius.

$C = \frac{5}{9}(F-32)$ 14°F

$C = \frac{5}{9}(14-32)$

$= \frac{5}{9}\left(-\frac{2}{18}\right)$

$= -10^\circ C$