Excellent +2

INTERMEDIATE ALGEBRA

GPS #1 1.1 Describing Data with Set of Numbers NAME: Paarul Patel

Properties of Real Numbers:

* Identity (0): The identity for addition is 0; $a+0=a$
* Identity (1): The identity for multiplication is 1; $a\cdot 1=a$
* Commutative: $a+b=b+a$ and $a\cdot b=b\cdot a$
* Associative: $(a+b)+c=a+(b+c)$ and $(a\cdot b)\cdot c=a\cdot(b\cdot c)$
* Distributive: $a(b+c)=ab+ac$ and $a(b-c)=ab-ac$

Note: a,b,c are real numbers.

20/20 Good job.

1. Complete the sets of numbers for the following:

a) Natural Numbers $=\{1,2,3,\ ,\ldots\}$

b) Whole Numbers $=\{0,1,2,3,\ldots\}$

c) Integers $=\{\ldots,\ -10,\ 1,\ 2,\ 3,\ldots\}$

2. Classify each real number as one or more of the following: natural number, whole number, integer, rational number or irrational number.

$\left(\underset{I}{-1},\frac{3}{2},0,\frac{7}{13},2,-10,-\sqrt{3},-\pi,8,\sqrt{9}=3\right)$ all Real number

Natural numbers: $2, 8, \sqrt{9}=3$

Whole numbers: $0, 2, 8, \sqrt{9}(3)$

Integers: $-1, -10, 0, 2, 8, 3$

Rational numbers: $\frac{3}{2}, \frac{7}{13}, -1, -10, 0, 2, 8, 3$

Irrational numbers: $-\sqrt{3}, -\pi$

3. State the property of real numbers that justifies the following statements.

a) $(2\cdot 4)\cdot x=2\cdot(4\cdot x)$ Associative x =

b) $3+p=p+3$ commutative

c) $(1\cdot 3)\cdot 6=3\cdot 6$ identity (multi)

d) $(3+2)+9=3+(2+9)$ Associative

4. Apply the distributive property to the following:

a) $2(x+5)=$ $2x+10$

b) $7x-3x+x=$ $x(7-3+1)$ (greatest common factor)
$=5x$

c) $8-3(x+3)=$ $8-3x-9$
$=-3x-1$

d) $5-2(x-2)=$ $5-2x+4$
$=9-2x$

5. Calculate the average of the list of numbers.

a) 2,5,11,6

$\frac{2+5+11+6}{4}=6$

b) 14,15,17,18,26

$\frac{14+15+17+18+26}{5}=18$

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GPS #2 **1.2 Operations on Real Numbers** **NAME:** Parul Patel

Some Useful Guidelines:

* Absolute Value $|a| = \begin{cases} a, a \geq 0 \\ -a, a < 0 \end{cases}$

* Additive Inverse: The additive inverse (or opposite) of a real number a is $-a$.

* Multiplicative Inverse: The multiplicative inverse (or reciprocal) of a real number a is $\frac{1}{a}$.

* Division: $\frac{a}{b} = a \cdot \frac{1}{b}$

Note: a, b, c are real numbers.

Evaluate the following:

1. a) $|-8|+3$
$8+3$
$=11$

b) $|-3|+|-2|+1$
$3+2+1$
$=6$

c) $|5|+|-7|+2$
$5+7+2$
$=14$

2 a) $|6-8+1|$
$=|-2+1|$
$=|-1|$
$=+1$

b) $|4.5-5+1|$
$\rightarrow |-.5+1|$
$\rightarrow |.5|$
$=0.5$

c) $|-2+3.2-5|$
$=|-3.8|$
$=+3.8$

3. Find the additive inverse, or opposite, of the following:

a) 347
-347

b) $-\frac{5}{6}$
$5/6$

c) $2-3x$
$-2+3x$

c) $-a+b$
$a-b$

4. Find the multiplicative inverse, or reciprocal, of the following:

a) $\frac{8}{9}$
$=\frac{9}{8}$

b) $-\frac{3}{4}$
$=-\frac{4}{3}$

c) $\frac{3}{x-1}$
$=\frac{x-1}{3}$

d) $4a$
$=1/4a$

5. Evaluate the following arithmetic operations and simplify.

a) $-2+1+(-3)+6$
$=2$

b) $\frac{-3}{2}-\left(\frac{-1}{2}\right)$
$=\frac{-3-(-1)}{2} = \frac{-3+1}{2} = -\frac{2}{2} = -1$

c) $-9 \div -3$
division is to
$=-9 \times -\frac{1}{3}$ as $(-9)(-\frac{1}{3})$
$=\frac{9}{3} = 3$

d) $\left(\frac{4}{3} \cdot \frac{3}{8}\right) \div \frac{1}{2}$
$=\left(\frac{1}{2}\right) \div \frac{1}{2} = \frac{1}{2} \times \frac{2}{1} = 1$

e) $7x-2x+x$
$=x(7-2+1)$
$=8x$

f) $\frac{1}{2}x-\frac{1}{4}x$
$=x\left(\frac{1 \cdot 2}{2 \cdot 2}-\frac{1}{4}\right)$
$=x\left(\frac{2-1}{4}\right)$
$=x(1/4) = \frac{x}{4}$

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GPS #3 **1.3 INTEGER EXPONENTS** **NAME:** Parul Patel

Useful Definitions and Rules: For all integers m and n and all real numbers a and b:

* Exponential Expression: $2^4 = 2\cdot2\cdot2\cdot2$ (4 factors of 2), $3^5 = 3\cdot3\cdot3\cdot3\cdot3$ (5 factors of 3), $a^5 = a\cdot a\cdot a\cdot a\cdot a$ (5 factors of a), $a^n = a\cdot a\cdot a\cdot a\cdot a...a$ (n factors of a)
* Product Rule: $a^m \cdot a^n = a^{m+n}$
* Zero Exponent: $a^0 = 1 (a \neq 0)$
* Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$
* Negative Exponent: $a^{-n} = \frac{1}{a^n} (a \neq 0)$
* Power Rules: $(a^m)^n = a^{mn}$; $(ab)^m = a^m b^m$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} (b \neq 0)$$

A number is in scientific notation when it is written as a product of a number between 1 and 10 (inclusive of 1) and an integer power of 10. For example: 2.35×10^{10}

Assume all variables represent nonzero real numbers for the following:

1. Evaluate the following:

a) $2^3 = 2\cdot2\cdot2 = 8$ b) $(-3)^4 = -3\cdot-3\cdot-3\cdot-3 = +9\cdot+9 = 81$ c) $-4^3 = -(4)(4)(4) = -64$

d) $\sqrt{25} = \sqrt{5^2} = (5^2)^{1/2} = 5$ e) $\sqrt[3]{8} = (2^3)^{1/3} = 2$ f) $\sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4^{2\cdot\frac{1}{2}}}{3^{2\cdot\frac{1}{2}}} = \frac{4}{3}$

2. Apply the product rule for exponents, if possible, in each case.

a) $4^5\cdot4^2 = 4^{(5+2)} = 4^7$ (Product Rule) b) $7^3\cdot7^9\cdot7^2 = 7^{(3+9+2)} = 7^{14}$

c) $(-4x^3)(6x^2) = -4\cdot x^3\cdot6\cdot x^2 = -24x^5$ d) $(2x^5)(y^2) = 2x^5y^2$

3. Apply the quotient rule for exponents and write each result using only positive exponents.

a) $\frac{2^5}{2^3} = 2^{(5-3)} = 2^2 = 4$ (Quotient Rule) b) $\frac{7^{-2}}{7^8} = 7^{(-2-8)} = 7^{-10} = \frac{1}{7^{10}}$

c) $\frac{x^8}{x^3} = x^{(8-3)} = x^5$ d) $\frac{y^4}{y^{-2}} = y^{(4-(-2))} = y^6$

3. Use one or more power rules to simplify each expression.

a) $(y^5)^2 = y^{10}$ (Power Rule) b) $(x^{-2})^3 = x^{-6} = \frac{1}{x^6}$

c) $(3x)^4 = 3^4\cdot x^4 = 81x^4$ d) $\left(\frac{6}{-5}\right)^2 = \frac{6^2}{-5^2} = \frac{36}{25}$

4. Evaluate and simplify each expression so that no negative exponents appear in the final result

a) $-(-7)^0 = -(1) = -1$ (Zero Exponent) b) $2x^0 - y^0 = 2(1) - 1 = 2-1 = 1$

c) $(2k^{-1})^4 = \left(\frac{2}{k^{+1}}\right)^4 = \frac{2^4}{k^4} = \frac{16}{k^4}$ (Negative Exponent) d) $\left(\frac{6x^{-2}}{x^{-3}}\right)^2 = \left(\frac{6x^3}{x^2}\right)^2 = (6x^{3-2})^2 = (6x^1)^2 = 36x^2$

5. Write each number in scientific notation.

a) $3600 = 3.6\times10^3$ b) $-790,000 = -7.9\times10^5$

c) $0.000896 = 8.96\times10^{-4}$ d) $-0.0000555 = -5.55\times10^{-5}$

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GPS #4 **1.4 VARIABLE, EQUATIONS, AND FORMULAS** **NAME:** Parul Patel

Useful Terminologies:

* Variable: Represents an unknown quantity. [Example: x, y, z, A, B, C.] can be any number $x=1$, $x=14$
* Algebraic Expression: Consists of numbers, variables, operation symbols, and grouping symbols. [Example: $3x-5$]
* Equation (=): A statement that two algebraic expressions are equal – contains an equal sign. [Example: $3x-5=y$]
* Formula: An equation used to calculate one quantity, using known values of other quantities. [Example: $P=2L+2W; A=\pi r^2; C=2\pi r$]

$y=ax$, to evaluate modal data

1. Write a formula for the following:

a) Find the area of a square with a side of x.

$A = x \cdot x$
$= x^2$

b) Find the area of a circle with a radius a.

$A = \pi r^2$
$= \pi a^2$

c) Find the circumference of a circle with a radius b.

$C = 2\pi r$
$C = 2\pi b$

2. Evaluate the formula for the given value(s) of the variable(s).

a) $P=2L+2W; L=3, W=4$

$P = 2(3)+2(4)$
$P = 6+8$
$P = 14$

b) $y=2x+1; x=8$

$y = 2(8)+1$
$= 16+1$
$y = 17$

c) $A=\frac{1}{2}bh; b=6, h=2$

$A = \frac{1}{2}(6)(2)$
$= \frac{1}{2}(12)$ $A = 6$

d) $y=x^2; x=5$

$y = 5^2$
$y = 25$

3. a) Find a value for a so that $y=ax$ models the data.

x	-2	-1	0	1	2
y	-8	-4	0	4	8

$y = ax$

$\frac{-8}{-2} = \frac{a(-2)}{-2}$ $a=4$

$\frac{-4}{-1} = \frac{a(-1)}{-1}$ $a=4$

$0 = a(0)$ $a=0$

$4 = a$

$y = 4x$

b) Find a value for a so that $y=ax$ models the data.

x	2	3	4	5	6
y	-4	-6	-8	-10	-12

$y = ax$

$\frac{-4}{2} = \frac{a2}{2}$ $a=-2$

$-8 = a4$ $a=-2$

$y = -2x$

4. a) Write an equation that models the data.

x	1	2	3	4	5
y	2	4	6	8	10

$y = ax$

$2 = a \times 1$ $a=2$

$6 = a3$ $a=2$

$y = 2x$

b) Write an equation that models the data.

x	-2	-1	0	1	2
y	4	1	0	1	4

$y = ax$

$4 = -2a$ $a=-2$

$1 = -a$ $a=-1$

$0 = a0$ $a=0$

$4 = 2a$ $a=2$

$y = x^2$

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GPS #5 **1.5 INTRODUCTION TO GRAPHING** **NAME:** Parul Patel

Useful Terminologies:

* Relation: A set of ordered pairs. [Example: $S = \{(-1,3),(2,4),(3,-2),(4,5)\}$]
* Domain: In a relation consisting of ordered pairs (x,y), the set of x-values is the domain.
* Range: In a relation consisting of ordered pairs (x,y), the set of y-values is the range.

1. Identify the domain and range of the following:

a) $S_1 = \{(0,4),(3,9),(6,-2),(4,5)\}$

$D = \{0, 3, 6, 4\}$

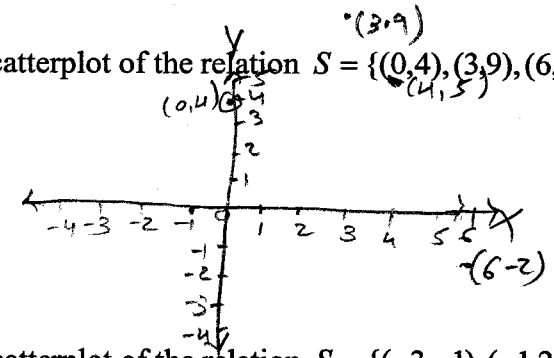
$R = \{4, 9, -2, 5\}$

b) $S_2 = \{(-1,2),(1,3),(5,-1),(9,2)\}$ Same members don't count

$D = \{-1, 1, 5, 9\}$

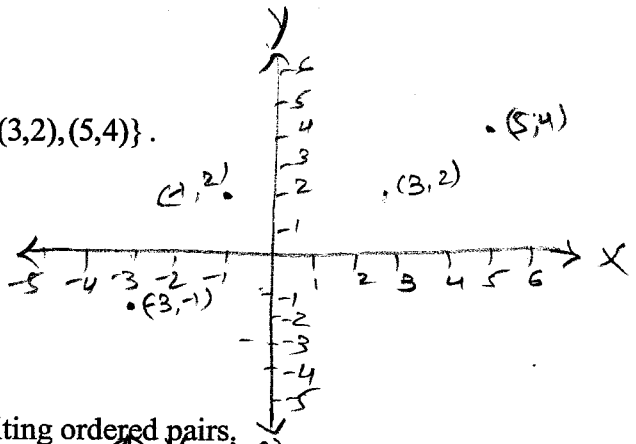
$R = \{2, 3, -1\}$

2. a) Make a scatterplot of the relation $S = \{(0,4),(3,9),(6,-2),(4,5)\}$. Label your points clearly



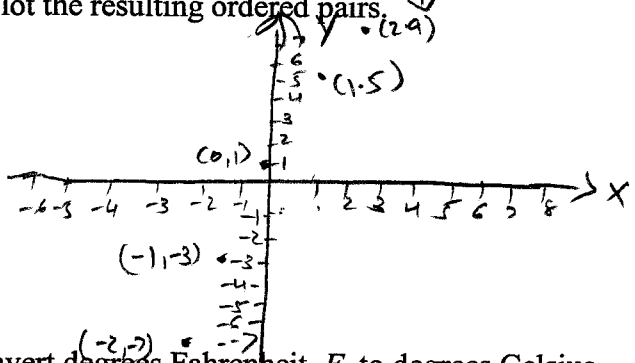
b) Make a scatterplot of the relation $S = \{(-3,-1),(-1,2),(3,2),(5,4)\}$.

Scatter Plot



3. Evaluate $y = 4x + 1$ for $x = -2, -1, 0, 1$ and 2. Plot the resulting ordered pairs.

x	y	(x,y)
-2	-7	(-2,-7)
-1	-3	(-1,-3)
0	1	(0,1)
1	5	(1,5)
2	9	(2,9)



4. The formula $C = \frac{5}{9}(F - 32)$ can be used to convert degrees Fahrenheit, F, to degrees Celsius, C. If the outside temperature is $14^\circ F$, find the equivalent temperature in Celsius.

$$C = \frac{5}{9}(F - 32)$$

$$= \frac{5}{9}(14 - 32)$$

$$= \frac{5}{9}(-18^{2})$$

$$\boxed{C = -10^{\circ}}$$