INTERMEDIATE ALGEBRA

GPS # 17 **5.1 POLYNOMIAL FUNCTIONS** **NAME:** Antoinette Durden

Useful Guidelines:

* A monomial is a term in which the variables have only nonnegative integer exponents. Example: $3x^2$, $5x^3y$, $-2x$, -7, and $3z^4y^5$.
* A polynomial is a term or a finite sum of terms in which all variables have whole number exponents and no variables appear in denominators. Example: $3x^2-4xy^2+1$.
* Adding polynomials is simply combining like terms together.
* Subtracting polynomials is simply adding the first polynomial and the negative of the second polynomial.

1. For each polynomial function, find $f(2)$ and $f(-3)$.

a) $f(x)=-4x+3$

$f(2)=-4(2)+3 \Rightarrow -8+3 \Rightarrow -5$

$f(-3)=-4(-3)+3 \Rightarrow 12+3 \Rightarrow 15$

b) $f(x)=x^2-2x+5$

$f(2)\ 2^2-2(2)+5 \Rightarrow 4-4+5 \Rightarrow 5$

$+(-3)\ -3^2-2(-3)+5 \Rightarrow 9+6+5 \Rightarrow 20$

Evaluate the following:

2. a) $(3x^2+7x-5)+(5x^2-2x+3)=$ $8x^2+5x-2$

b) $(y^3-2y+1)+(y^3-8y-13)=$ $2y^3-10y-12$

c) $(-z^4+7z^2-2z)+(2z^3-2z^2+3z)=$ $-z^4+2z^3+5z^2+z$

d) $(5m^3-2m+1)+(-m^3+8m^2-15)=$ $4m^3+8m^2-2m-14$

3. a) $(9r^2-7r+6)-(5r^2+2r-2)=$ $9r^2-7r+6-5r^2-2r+2=4r^2-9r+8$

b) $(t^3+3t-2)-(t^3-8t-3)=$ $t^3+3t-2-t^3+8t+3=11t+1$

c) $(-z^4-4z^2+8z)-(3z^3+9z^2-z)=$ $-z^4-4z^2+8z-3z^3-9z^2+z=-z^4-3z^3-13z^2+9z$

d) $(9m^3-m-10)-(-2m^3+3m^2-5)=$ $9m^3-m-10+2m^3-3m^2+5=11m^3-3m^2-m-5$

e) $(10n^4-n^2+2p)-(-2n^4+n^2-3p)-(n^4-2n^2+p)=$ $11n^4+4p$

$10n^4-n^2+2p+2n^4-n^2+3p-n^4+2n^2-p$

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GPS # 18 **5.2 MULTIPLICATIONS OF POLYNOMIALS** **NAME:** Antoinette Durden

Useful Definitions:

* To multiply polynomials, simply multiply each term of the first polynomial to each term of the other. Example: $(x^2+2x)(7x^2-5x+3) = x^2(7x^2-5x+3)+2x(7x^2-5x+3)$

* **Special Products:**

$(x+y)(x-y) = x^2-y^2$

$(x+y)^2 = x^2+2xy+y^2$

$(x-y)^2 = x^2-2xy+y^2$

Note:

$(x+y)^2 \neq x^2+y^2$

$(x-y)^2 \neq x^2-y^2$

Evaluate the following:

a) $(x^2+3x)(2x^2+3x+1)$

$x^2(2x^2+3x+1) + 3x(2x^2+3x+1)$

$2x^4+3x^3+x^2 + 6x^3+9x^2+3x$

$2x^2+9x^3+10x^2+3x$

b) $(2m+3)(m^2-3m-4)$

$2m(m^2-3m-4)+3(m^2-3m-4)$

$2m^3-6m^2-8m+3m^2-9m-12$

$2m^3-3m^2-17m-12$

c) $(2t-3)(4t-1)$

$2t(4t-1)-3(4t-1)$

$8t^2-2t-12t+3$

$8t^2-14t+3$

d) $(a-3)(a+3)$

$a(a+3)-3(a+3)$

$a^2+3a-3a-9$

a^2-9

e) $(x-5)(x-5)$

$x(x-5)-5(x-5)$

$x^2-5x-5x+25$

$x^2-10x+25$

f) $(2p+8)(2p+8)$

$2p(2p+8)\ 8(2p+8)$

$4p^2+16p+16p+64$

$4p^2+32p+64$

g) $(x+2)(x-2)$

$x(x-2)\ 2(x-2)$

$x^2-2x+2x-4$

x^2-4

h) $(3x-2)(x-4)$

$3x(x-4)-2(x-4)$

$3x^2-12x-2x+8$

$3x^2-14x+8$

i) $(x+2)^2$

$(x+2)(x+2)$

$x(x+2)\ 2(x+2)$

$x^2+2x+2x+4$

x^2+4x+4

j) $(3x-2)^2$

$(3x-2)(3x-2)$

$3x(3x-2)-2(3x-2)$

$9x^2-6x-6x+4$

$9x^2-12x+4$

k) $(3-5x)^2$

$(3-5x)(3-5x)$

$3(3-5x)-5x(3-5x)$

$9-15x-15x+25x^2$

$25x^2-30x+9$

l) $-2x(3x-2)^2$

$-2x(9x^2-12x+4)$

$-18x^3+24x^2-8x$

~~$6x^2-8x$~~

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GPS # 19 **5.3 FACTORING POLYNOMIALS** **NAME:** Antoinette Durden

Useful Guidelines:

Greatest Common Factor: The largest common term that can be factor out from the polynomial.

For example: $3xy^2 + 6x = 3x(y^2 + 2)$, where $3x$ is the Greatest Common Factor.

Factoring by grouping:

Step 1: Group the terms so that each group has a common factor.
For example: $6x - 6y + 2x - 2y = (6x - 6y) + (2x - 2y)$

Step 2: Factor out the common factor in each group.
For example: $(6x - 6y) + (2x - 2y) = 6(x - y) + 2(x - y)$

Step 3: Factor out the common factor from the groups, if possible. Otherwise, try a different grouping. For example: $6(x - y) + 2(x - y) = (x - y)(6 + 2) = 8(x - y)$

1. Factor out the greatest common factor. Simplify the factors, if possible.

a) $x^2y - 3xy =$ $xy(x-3)$

b) $6p^2q^3 - 12pq =$ $6pq(pq^2-2)$

c) $(p-2)(p+2) - (p+4)(p+2) =$ $(p+2)[(p-2)-(p+4)] \Rightarrow (p+2)[p-2-p-4]$
$(p+2)(-6) =$ $-6p-12$

d) $(x-3)(y+2) - (x+4)(y+2) =$ $(y+2)[(x-3)-(x+4)] \Rightarrow (y+2)[x-3-x+4] =$
$(y+2)(-7) =$ $-7y-14$

2. Factor by grouping and simplify the factors, if possible.

a) $3x + 3y + 7x + 7y =$ $3(x+y) + 7(x+y) \Rightarrow (x+y) + (3+7) \Rightarrow 10(x+y) \Rightarrow$
$10x+10y$

b) $15a + 3n + 5ab + nb =$ $3(5a+n) + b(5a+n) \Rightarrow (5a+n)(3+b)$

c) $30 + 5x + 18y + 3xy =$ $5(6+x) + 3y(6+x) \Rightarrow (6+x)(5+3y)$

d) $4m^3 + m^2 - 24m - 6$ $m^2(4m+1) + 6(4m+1) \Rightarrow (4m+1)(m^2-6)$

e) $2ab - 2b + 1 - a$ $2b(a-1) + 1(a-1) \Rightarrow (a-1)(2b-1)$

3. Solve the equation.

a) $6y^2 + 3y = 0$ $3y(2y+1) = 0 \Rightarrow 3y = 0$ or $(2y+1) = 0$
$3y = 0$ $2y + 1 = 0$ $y = -\frac{1}{2}$ $\{y \mid 0, -\frac{1}{2}\}$
$y = 0$ $2y = -1$

b) $8x^3 = 2x^2$
$8x^3 - 2x^2 = 0 =$ $2x^2(4x-1) = 0$ $2x^2 = 0$ or $(4x-1) = 0$
$2x^2 = 0$ $4x = 1$
$x = 0$ $x = \frac{1}{4}$ $\{x \mid 0, \frac{1}{4}\}$

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GPS # 20 **5.4 FACTORING TRINOMIALS** **NAME:** Antoinette Durden

Useful Guidelines:

To factor ax^2+bx+c:

Step 1: Find pair whose product is ax^2.
For example: $3x^2+7x+2=(3x+...)(x+...)$

Step 2: Find pair whose product is c.
For example: $3x^2+7x+2=(3x+2)(x+1)$? or $3x^2+7x+2=(3x+1)(x+2)$?

Step 3: Check which one will produce the middle term bx. That's the right factoring.
If there is no such middle term, the polynomial cannot be factored; it is called prime.

20/20

Factor each polynomial.

a) $x^2+3x+2=$
$(x+1)(x+2)$

b) $p^2-4p+3=$
$(p-1)(p-3)$

c) $2x^2+7x+3=$
$(2x+1)(x+3)$

d) $3y^2+13y+4=$
$(3y+1)(y+4)$

e) $3r^2+14r+5=$ Prime
$(3r-1)(r+5)$

f) $x^2-3x+2=$
$(x-1)(x-2)$

g) $3m^2+12m-15=$
$\frac{(3m-3)(m+5)}{3(m^2+4m-5)}$
$3(m-1)(m+5)$

h) $-12y^3-8y^2+4y=$
$\frac{(-12y^2+4)(y+y)}{-4y(3y^2+2y-1)}$
$-4y(3y-1)(y+1)$

i) $6a^2+5ab-4b^2=$
$(3a+4b)(2a-b)$

j) $2z^4-8z^2+6=$
$\frac{(2z^2-6)(z^2-1)}{2(z^4-4z^2+3)}$
$2(z^2-1)(z^2-3)$

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GPS # 21 **5.5 SPECIAL TYPES OF FACTORING** **NAME:** Antoinette Durden

Useful Types of Factoring:

1. Difference of Square: $x^2 - y^2 = (x-y)(x+y)$
2. Perfect Square Trinomial: $x^2 + 2xy + y^2 = (x+y)^2$

   $x^2 - 2xy + y^2 = (x-y)^2$
3. Difference of Cubes: $x^3 - y^3 = (x-y)(x^2+xy+y^2)$
4. Sum of Cubes: $x^3 + y^3 = (x+y)(x^2-xy+y^2)$

w/ hw

Factor each polynomial.

1. a) $x^2 - 9 =$

$(x-3)(x+3)$

b) $4p^2 - 64 =$

$= (2p)^2 - (8)^2$

$= (2p-8)(2p+8)$ conjugate

2. a) $9x^2 + 12x + 4 =$

$= (3x)^2 + 2(3x)(2) + (2)^2$

$= (3x+2)(3x+2)$

$= (3x+2)^2$

b) $16p^2 - 40pm + 25m^2 =$

$= (4p-5m)(4p-5m)$

$= (4p-5m)^2$

3. a) $8r^3 - 27 =$

$= (2r)^3 - (3)^3$

$= (2r-3)(4r^2 + 6r + 9)$

b) $x^3 - 1000y^3 =$

$= (x)^3 - (10y)^3$

$= (x-10y)(x^2 + 10xy + 100y^2)$

4. a) $27t^3 + 64 =$

$= (3t)^3 + (4)^3$

$= (3t+4)(9t^2 - 12t + 16)$

b) $(z+1)^3 + y^3 =$ $(z+1+y)[(z+1)^2 - (z+1)(y) + y^2]$

$(z+1+y)[(z+1)^2 - (z+1)(y) + y^2]$

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GPS # 22 **5.5 SPECIAL TYPES OF FACTORING II** **NAME:** Antoinette Darden

Useful Types of Factoring:

1. Difference of Square: $x^2 - y^2 = (x-y)(x+y)$
2. Perfect Square Trinomial: $x^2 + 2xy + y^2 = (x+y)^2$
   $x^2 - 2xy + y^2 = (x-y)^2$
3. Difference of Cubes: $x^3 - y^3 = (x-y)(x^2+xy+y^2)$
4. Sum of Cubes: $x^3 + y^3 = (x+y)(x^2-xy+y^2)$

Hints: To factor a polynomial,

a. Factor out any common factor

b. If the polynomial is a binomial: 1,3,4

c. If the polynomial is a trinomial: 2

d. If the polynomial is more than 3 terms, try "factoring by grouping"

Factor each polynomial.

1. a) $3x(4+b) - 2x(4+b) =$

$= (4+b)(3x-2x)$

$= (4+b)x$

b) $4pq^2 - 16p^2q =$

$4pq(q - 4p)$

2. a) $9x^2 - 64y^2 =$

$= (3x)^2 - (8y)^2$

$= (3x-8y)(3x+8y)$

b) $64p^3 - 1000 =$

$= (4p)^3 - (10)^3$

$= (4p-10)(16p^2+40p+100)$

3. a) $9r^2 - 24r + 16 =$

$= (3r)^2 - (4)^2$

$= (3r-4)^2$

b) $4t^2 + 12t + 9 =$

$= (2t)^2 + (3)^2$

$= (2t+3)^2$

4. a) $(m^3 - 5mn^2) + (m^2n - 5n^3) =$

$= m(m^2 - 5n^2) + n(m^2 - 5n^2)$

$= (m^2 - 5n^2)(m+n)$

b) $(12x^3 + 4x^2) - (27x - 9) =$

$= 4x^2(3x+1) - 9(3x+1)$

$= (3x+1)(4x^2 - 9)$

$= (3x+1)(2x+3)(2x-3)$

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GPS # 23 **5.6 POLYNOMIAL EQUATIONS** **NAME:** Antoinette Durden

Useful Guidelines:

To Solve a Quadratic Equation, $2x^2+x-3=0$

1. Factor the polynomial: $(2x+3)(x-1)=0$
2. Set each variable factor equal to zero: $(2x+3)=0$ or $(x-1)=0$
4. Find the solution(s): $x=-\frac{3}{2}$ or $x=1 \Rightarrow$ the solution set is $\{-\frac{3}{2},1\}$

Zero-Factor Property:
If two numbers have a product of **0**, then at least one of the numbers must be **0**.

Solve the following equations:

1. a) $x^2+4x-5=0$

$(x+1)(x+5)=0$

$x-1=0$ or $x+5=0$

$x=1$ $\quad x=-5$

$(1)^2+(4)(1)-5=0$ $\quad (-5)^2+4(-5)-5=0$

$1+4-5$ $\quad 25-20-5=0$

$5-5=0$ $\quad 25-25=0$

$\{x \mid x=-5, x=1\}$

b) $3p^2-11p+6=0$

$(3p-2)(p-3)=0$

$3p-2=0$ or $p-3=0$

$3p=2$ $\quad p=3$

$p=\frac{2}{3}$

$\{\frac{2}{3},3\}$

Sol set $\{p \mid p=\frac{2}{3}$ or $p=3\}$

2. a) $5z^2-12z=-7$

$5z^2-12z+7=0$

$(5z-7)(z-1)=0$

$5z-7=0$ or $z-1=0$

$5z=7$ $\quad z=1$

$z=\frac{7}{5}$

$\{z \mid z=\frac{7}{5}$ or $z=1\}$

b) $7k^2-23k=-6$

$7k^2-23k+6=0$

$(7k-2)(k-3)=0$

$7k-2=0$ or $k-3=0$

$7k=2$ $\quad k=3$

$k=\frac{2}{7}$

$\{k \mid k=\frac{2}{7}$ or $k=3\}$

3. a) $4x^3+4x^2-8x=0$

$4x(x^2+x-2)=0$

$4x(x+2)(x-1)=0$

$4x=0$ or $x+2=0$ or $x-1=0$

$x=0$ $\quad x=-2$ $\quad x=1$

$\{x \mid x=-2$ or $x=0$ or $x=1\}$

b) $(y^3-2y^2)-(4y+8)=0$

[Factor by grouping]

$\Rightarrow y^2(y-2)-4(y-2)=0$

$\Rightarrow (y-2)(y^2-4)=0$

$\Rightarrow (y-2)(y+2)(y-2)=0$

$y-2=0$ $\quad y+2=0$

$y=2$ $\quad y=-2$

$\{y \mid y=-2$ or $y=2\}$