INTERMEDIATE ALGEBRA

GPS # 17 **5.1 POLYNOMIAL FUNCTIONS** **NAME:** Parul Patel

Useful Guidelines:

* A monomial is a term in which the variables have only nonnegative integer exponents. Example: $3x^2$, $5x^3y$, $-2x$, -7, and $3z^4y^5$.
* A polynomial is a term or a finite sum of terms in which all variables have whole number exponents and no variables appear in denominators. Example: $3x^2 - 4xy^2 + 1$.
* Adding polynomials is simply combining like terms together.
* Subtracting polynomials is simply adding the first polynomial and the negative of the second polynomial.

Good Job!

1. For each polynomial function, find $f(2)$ and $f(-3)$.

a) $f(x) = -4x + 3$

$f(2) = -4(2) + 3$
$= -8 + 3$
$= -5$

$f(-3) = -4(-3) + 3$
$= 12 + 3$
$= 15$

b) $f(x) = x^2 - 2x + 5$

$= (2)^2 - 2(2) + 5$
$= 4 - 4 + 5$
$= 5$

$= (-3)^2 - 2(-3) + 5$
$= 9 + 6 + 5$
$= 20$

Evaluate the following: Polynomial

2. a) $(3x^2 + 7x - 5) + (5x^2 - 2x + 3) = 8x^2 + 5x - 2$

b) $(y^3 - 2y + 1) + (y^3 - 8y - 13) = 2y^3 - 10y - 12$

c) $(-z^4 + 7z^2 - 2z) + (2z^3 - 2z^2 + 3z) = -z^4 + 2z^3 + 5z^2 + z$

d) $(5m^3 - 2m + 1) + (-m^3 + 8m^2 - 15) = 4m^3 + 8m^2 - 2m - 14$

3. a) $(9r^2 - 7r + 6) - (5r^2 + 2r - 2) = 9r^2 - 7r + 6 - 5r^2 - 2r + 2$
$= 4r^2 - 9r + 8$

b) $(t^3 + 3t - 2) - (t^3 - 8t - 3) = t^3 + 3t - 2 - t^3 + 8t + 3$
$= 11t + 1$

c) $(-z^4 - 4z^2 + 8z) - (3z^3 + 9z^2 - z) = -z^4 - 4z^2 + 8z - 3z^3 - 9z^2 + z$
$= -z^4 - 3z^3 - 13z^2 + 9z$

d) $(9m^3 - m - 10) - (-2m^3 + 3m^2 - 5) = 9m^3 - m - 10 + 2m^3 - 3m^2 + 5$
$= 11m^3 - 3m^2 - m - 5$

e) $(10n^4 - n^2 + 2p) - (-2n^4 + n^2 - 3p) - (n^4 - 2n^2 + p) = 10n^4 - n^2 + 2p + 2n^4 - n^2 + 3p - n^4 + 2n^2 - p$
$= 11n^4 + 4p - 5p$

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GPS # 18 **5.2 MULTIPLICATIONS OF POLYNOMIALS** **NAME:** Parul Patel

Useful Definitions:

* To multiply polynomials, simply multiply each term of the first polynomial to each term of the other. Example: $(x^2+2x)(7x^2-5x+3) = x^2(7x^2-5x+3)+2x(7x^2-5x+3)$

* **Special Products:**

$(x+y)(x-y) = x^2-y^2$

$(x+y)^2 = x^2+2xy+y^2$

$(x-y)^2 = x^2-2xy+y^2$

Note:

$(x+y)^2 \neq x^2+y^2$

$(x-y)^2 \neq x^2-y^2$

Evaluate the following:

a) $(x^2+3x)(2x^2+3x+1)$

$= x^2(2x^2+3x+1)+3x(2x^2+3x+1)$

$= 2x^4+3x^3+x^2+6x^3+9x^2+3x$

$= 2x^4+9x^3+10x^2+3x$

b) $(2m+3)(m^2-3m-4)$

$= 2m(m^2-3m-4)+3(m^2-3m-4)$

$= 2m^3-6m^2-8m+3m^2-9m-12$

$= 2m^3-3m^2-17m-12$

c) $(2t-3)(4t-1)$

$= 2t(4t-1)-3(4t-1)$

$= 8t^2-2t-12t+3$

$= 8t^2-14t+3$

see other method in back

d) $(a-3)(a+3)$

$= a(a+3)-3(a+3)$

$= a^2+3a-3a-9$

$= a^2-9$

e) $(x-5)(x-5)$

$= x(x-5)-5(x-5)$

$= x^2-5x-5x+25$

$= x^2-10x+25$

f) $(2p+8)(2p+8)$ Inner to Inner & (Foil) outer to outer method

$= 4p^2+16p+16p+64$

$= 4p^2+32p+64$

g) $(x+2)(x-2)$ (Foil)

x^2-4

h) $(3x-2)(x-4)$ (Foil)

$= 3x^2-12x-2x+8$

$= 3x^2-14x+8$

i) $(x+2)^2 \Rightarrow (x+2)(x+2)$

x^2+4x+4

j) $(3x-2)^2 \Rightarrow (3x-2)(3x-2)$

$9x^2-12x+4$

k) $(3-5x)^2$

$= 9-30x+25x^2$

l) $-2x(3x-2)^2$

$= -2x(9x^2-12x+4)$

$= 18x^3+24x^2-8x$

all done

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GPS # 19 **5.3 FACTORING POLYNOMIALS** **NAME:** Parul Patel

Useful Guidelines:

Greatest Common Factor: The largest common term that can be factor out from the polynomial.

For example: $3xy^2 + 6x = 3x(y^2 + 2)$, where $3x$ is the Greatest Common Factor.

Factoring by grouping:

Step 1: Group the terms so that each group has a common factor.
For example: $6x - 6y + 2x - 2y = (6x - 6y) + (2x - 2y)$

Step 2: Factor out the common factor in each group.
For example: $(6x - 6y) + (2x - 2y) = 6(x - y) + 2(x - y)$

Step 3: Factor out the common factor from the groups, if possible. Otherwise, try a different grouping. For example: $6(x - y) + 2(x - y) = (x - y)(6 + 2) = 8(x - y)$

1. Factor out the greatest common factor. Simplify the factors, if possible.

a) $x^2y - 3xy =$ $xy(x - 3)$

b) $6p^2q^3 - 12pq =$ $6pq(pq^2 - 2)$

c) $(p-2)(p+2) - (p+4)(p+2) =$ $(p+2)[(p-2) - (p+4)]$
$= (p+2)[p - 2 - p - 4]$
$= (p+2)[-6]$ $= -6p - 12$

d) $(x-3)(y+2) - (x+4)(y+2) =$
$= (y+2)[(x-3) - (x+4)]$
$= (y+2)[x - 3 - x - 4]$
$= (y+2)[-7]$ $= -7y - 14$

2. Factor by grouping and simplify the factors, if possible.

a) $3x + 3y + 7x + 7y =$ $3(x+y) + 7(x+y)$
$= (x+y)(3+7)$
$= (x+y)(10)$ $10x + 10y$

b) $15a + 3n + 5ab + nb =$ $3(5a+n) + b(5a+n)$
$= (5a+n)(3+b)$

c) $30 + 5x + 18y + 3xy =$ $5(6+x) + 3y(6+x)$
$= (6+x)(5+3y)$

d) $4m^3 + m^2 - 24m - 6 =$ $m^2(4m+1) - 6(4m+1)$
$= (4m+1)(m^2 - 6)$

e) $2ab - 2b + 1 - a =$ $2b(a-1) - 1(a-1)$
$= (a-1)(2b-1)$

3. Solve the equation.

a) $6y^2 + 3y = 0$ $3y(2y+1) = 0$
$3y = 0$ or $2y + 1 = 0$
$y = 0$ $y = -\frac{1}{2}$
Sol. set $\{y \mid 0, -\frac{1}{2}\}$ or $\{0, -\frac{1}{2}\}$

b) $8x^3 = 2x^2$ $8x^3 - 2x^2 = 0$
$2x^2(4x - 1) = 0$
$2x^2 = 0$ or $4x - 1 = 0$
$x = 0$ $4x = 1$
$x = \frac{1}{4}$
Sol. set $\{x \mid 0, \frac{1}{4}\}$ or $\{0, \frac{1}{4}\}$

all done

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GPS # 20 **5.4 FACTORING TRINOMIALS** **NAME:** Paarul Patel

Good Work!

Useful Guidelines:

To factor ax^2+bx+c:

Step 1: Find pair whose product is ax^2.
For example: $3x^2+7x+2=(3x+\ldots)(x+\ldots)$

Step 2: Find pair whose product is c.
For example: $3x^2+7x+2=(3x+2)(x+1)$? or $3x^2+7x+2=(3x+1)(x+2)$?

Step 3: Check which one will produce the middle term bx. That's the right factoring.
If there is no such middle term, the polynomial cannot be factored; it is called prime.

Factor each polynomial.

a) $x^2+3x+2=$ 1st term is same so 1&2 2nd term don't matter
$(x+1)(x+2)$
$+x$, $+2x$, needs $+3x$

b) $p^2-4p+3=$
$(p-3)(p-1)$
$-3p$, $-p$, needs $-4p$

c) $2x^2+7x+3=$ 1st term is not same so 1&3 (2nd term) does matter check both 1 at time
$(2x+1)(x+3)$
$+1x$, $+6x$, needs $+7x$

d) $3y^2+13y+4=$
$(3y+1)(y+4)$
y, $12y$

Guess all Possible
$(3y+2)(y+2)$
$2y$, $6y$ not possible
So change 2nd term

e) $3r^2+14r+5=$
5, 1
$(3r-1)(r+5)$
$-1r$, $+15r$
eval: $3r^2-5-r-5 \neq 3r^2+14r+5$
Not equal: So it calls (PRIME)
(com'n factor)

f) $x^2-3x+2=$ check by eyem
$(x-1)(x-2)$
make sure to evaluate Back to get Solut

g) $3m^2+12m-15=$
$3(m^2+4m-5)$
$3(m+5)(m-1)$
$+5m$, $-m$, need $+4m$

h) $-12y^3-8y^2+4y=$
$=-4y(3y^2+2y-1)$
$=-4y(3y-1)(y+1)$
$-y$, $3y$ needs $+2y$

i) $6a^2+5ab-4b^2=$
4b 8b
b 4b
$(3a+4b)(2a-b)$
$8ab$, $-3ab$

j) $2z^4-8z^2+6=$
$2(z^4-4z^2+3)$
$2(z^2-3)(z^2-1)$
$-3z^2$, $-z^2$

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GPS # 21 **5.5 SPECIAL TYPES OF FACTORING** **NAME:** Parul Patel

Useful Types of Factoring:

1. Difference of Square: $x^2 - y^2 = (x-y)(x+y)$
2. Perfect Square Trinomial: $x^2 + 2xy + y^2 = (x+y)^2$

   $x^2 - 2xy + y^2 = (x-y)^2$
3. Difference of Cubes: $x^3 - y^3 = (x-y)(x^2+xy+y^2)$
4. Sum of Cubes: $x^3 + y^3 = (x+y)(x^2-xy+y^2)$

(x+y) (x-y)

Conjugate of each oth

Factor each polynomial.

1. a) $x^2 - 9 =$ $x^2 - 3^2$

   $(x-3)(x+3)$

   b) $4p^2 - 64 =$ $(2p)^2 - (8)^2$

   $= (2p-8)(2p+8)$

2. a) $9x^2 + 12x + 4 =$ $(3x)^2 + 12x + 4$

   $(3x+2)^2$

   b) $16p^2 - 40pm + 25m^2 =$

   $(4p - 5m)^2$

3. a) $8r^3 - 27 =$ $(2r)^3 - (3)^3$

   $(2r-3)(4r^2+6r+9)$

   b) $x^3 - 1000y^3 =$ $(x)^3 - (10y)^3$

   $(x-10y)(x^2+10xy+100y^2)$

4. a) $27t^3 + 64 =$ $(3t)^3 + (4)^3$

   $(3t+4)(9t^2-12t+16)$

   b) $(z+1)^3 + y^3 =$

   $(z+1+y)((z+1)^2 - (z+1)(y) + y^2)$

$(z+1)\cdot y$

$zy + y$

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GPS # 22 **5.5 SPECIAL TYPES OF FACTORING II** **NAME:** Parul Patel

Useful Types of Factoring:

1. Difference of Square: $x^2 - y^2 = (x-y)(x+y)$
2. Perfect Square Trinomial: $x^2 + 2xy + y^2 = (x+y)^2$
   $x^2 - 2xy + y^2 = (x-y)^2$
3. Difference of Cubes: $x^3 - y^3 = (x-y)(x^2+xy+y^2)$
4. Sum of Cubes: $x^3 + y^3 = (x+y)(x^2-xy+y^2)$

Hints: To factor a polynomial,

a. Factor out any common factor

b. If the polynomial is a binomial: 1,3,4

c. If the polynomial is a trinomial: 2

d. If the polynomial is more than 3 terms, try "factoring by grouping"

Factor each polynomial.

1. a) $3x(4+b) - 2x(4+b) =$

$(4+b)(3x-2x)$

$(4+b)\,x$

$4x + bx$

b) $4pq^2 - 16p^2q =$ $4pq(q-4p)$

2. a) $9x^2 - 64y^2 =$ $(3x)^2 - (8y)^2$

$(3x-8y)(3x+8y)$

b) $64p^3 - 1000 =$ $(4p)^3 - (10)^3$

$(4p-10)(16p^2+40p+100)$

3. a) $9r^2 - 24r + 16 =$ $(3r)^2 \quad (4)^2$

$(3r-4)^2$

b) $4t^2 + 12t + 9 =$

$(2t+3)^2$

4. a) $m^3 - 5mn^2 + m^2n - 5n^3 =$

$m(m^2-5n^2) + n(m^2-5n^2)$

$(m^2-5n^2)(m+n)$

b) $12x^3 + 4x^2 - 27x - 9 =$

$4x^2(3x+1) - 9(3x+1)$

$(3x+1)(4x^2-9)$

$(3x+1)(2x-3)(2x+3)$

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GPS # 23 **5.6 POLYNOMIAL EQUATIONS** NAME: Parul Patel

Useful Guidelines:

To Solve a Quadratic Equation, $2x^2 + x - 3 = 0$

1. Factor the polynomial: $(2x+3)(x-1) = 0$
2. Set each variable factor equal to zero: $(2x+3) = 0$ or $(x-1) = 0$
4. Find the solution(s): $x = -\frac{3}{2}$ or $x = 1 \Rightarrow$ the solution set is $\{-\frac{3}{2}, 1\}$

Zero-Factor Property:

If two numbers have a product of **0**, then at least one of the numbers must be **0**.

Solve the following equations:

1. a) $x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$5x - x = 4x$

$x+5=0$ or $x-1=0$

$x = -5$ $\quad x = 1$

Sol. set $\{-5, 1\}$

b) $3p^2 - 11p + 6 = 0$

$(3p-2)(p-3) = 0$

$-2p - 9p = -11p$

$3p-2=0$ or $p-3=0$

$p = \frac{2}{3}$ $\quad p = 3$

Sol. set $\{\frac{2}{3}, 3\}$

2. a) $5z^2 - 12z = -7$

$5z^2 - 12z + 7 = 0$

$(5z-7)(z-1) = 0$

$-7z - 5z = -12z$

$5z - 7 = 0$ or $z - 1 = 0$

$z = \frac{7}{5}$ $\quad z = 1$

Sol. set $\{\frac{7}{5}, 1\}$

b) $7k^2 - 23k = -6$

$7k^2 - 23k + 6 = 0$

$(7k-2)(k-3) = 0$

$-2k - 21k = -23k$

$7k - 2 = 0$ or $k - 3 = 0$

$k = \frac{2}{7}$ $\quad k = 3$

Sol. set $\{\frac{2}{7}, 3\}$

3. a) $4x^3 + 4x^2 - 8x = 0$

$4x(x^2 + x - 2) = 0$

$4x(x+2)(x-1) = 0$

$4x = 0$ $\quad x+2=0$ $\quad x-1=0$

$x = 0$ $\quad x = -2$ $\quad x = 1$

Sol. set $\{-2, 0, 1\}$

b) $y^3 - 2y^2 - 4y + 8 = 0$

[Factor by grouping]

$y^2(y-2) - 4(y-2) = 0$

$(y-2)(y^2-4)$

$(y-2)(y-2)(y+2)$

$y-2=0$ $\quad y+2=0$

$y = 2$ $\quad y = -2$

Sol. set $\{-2, 2\}$