INTERMEDIATE ALGEBRA

GPS # 24 **6.1 RATIONAL FUNCTIONS AND EQUATIONS** **NAME:** Antoinette Durden

Good Job!

Useful Guidelines:

* Fundamental Property of Rational Numbers

  If $\frac{a}{b}$ is a rational number and if c is any nonzero real number, then $\frac{a}{b} = \frac{ac}{bc}$.

  (The numerator a and the denominator b above may be multiplied and divided by the same nonzero number c without changing the value of the rational number $\frac{a}{b}$.)

* Writing a Rational Expression in Lowest Terms: Factor both numerator and denominator to find their Greatest Common Factor and apply the Fundamental Property.

1. Evaluate $f(x)$ numerically for the given value of x.

   a) $f(x) = \frac{5x}{x^2 - 25}, x = 3$

   $= \frac{5(3)}{3^2 - 25} = \frac{15}{9 - 25} = -\frac{15}{16}$

   fb) $f(x) = \frac{x}{36 - x^2}, x = 6$

   $= \frac{6}{36 - 6^2} = \frac{6}{36 - 36} = \frac{6}{0}$ undefined

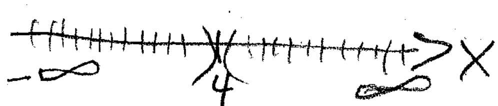
2. Write each rational expression in lowest terms.

   a) $\frac{x-2}{2x^2 - 8} = \frac{x-2}{2(x^2-4)} = \frac{\cancel{x-2}}{2(\cancel{x-2})(x+2)} = \frac{1}{2(x+2)}$ "lowest term"

   b) $\frac{2x+6}{(x-3)(x+3)} = \frac{2(\cancel{x+3})}{(x-3)(\cancel{x+3})} = \frac{2}{x-3}$

3. Find the domain of the rational function.

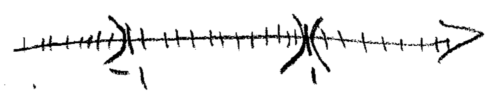
   a) $f(x) = \frac{x+4}{x^2 - 16} = \frac{\cancel{x+4}}{(x-4)(\cancel{x+4})} = \frac{1}{x-4}$

   $D: (-\infty, 4) \cup (4, \infty)$

   $D: \{x \mid x \neq 4\}$

   b) $f(x) = \frac{x-7}{x^2 - 1} = \frac{x-7}{(x+1)(x-1)}$

   $D: (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

   $D: \{x \mid x \neq -1 \text{ or } x \neq 1\}$

* 4. Solve the rational equation.

   a) $\frac{x}{x-5} + 1 = -6 = \frac{x}{x-5} = -7 =$

   $(x-5)\left[\frac{x}{\cancel{x-5}}\right] = [-7](x-5)$

   $x = -7x + 35$

   $\frac{8x}{8} = \frac{35}{8} = x = \frac{35}{8}$

   b) $(\cancel{x+2})\frac{x-1}{\cancel{x+2}} = \frac{6x}{\cancel{x+2}}(\cancel{x+2}) =$

   $x - 1 = 6x$

   $-1 = 5x$

   $-\frac{1}{5} = x$

   * "solve for least common denominator"

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GPS # 25   6.2 MULTIPLICATION AND DIVISION OF RATIONAL EXPRESSIONS   NAME:

2 w/o Good Job!

Useful Guidelines: [a, b, c, and d are nonzero real number.]

* Simplifying Rational Expressions: $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$. Example: $\frac{3b(c-3)}{5(c-3)} = \frac{3b}{5}$
* Multiplying Rational Expressions: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. Example: $\frac{(b+3)}{(a+2)} \cdot \frac{(a+7)}{(b+1)} = \frac{(b+3)(a+7)}{(a+2)(b+1)}$
* Dividing Rational Expressions: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$. Example: $\frac{3a}{5b} \div \frac{(a+2)}{(b+3)} = \frac{3a(b+3)}{5b(a+2)}$
* Writing a Rational Expression in Lowest Terms: Factor both numerator and denominator to find their Greatest Common Factor and apply the Fundamental Property.

1. Write each rational expression in lowest terms.

a) $\frac{33b^3}{3b} = \frac{\cancel{3b}(11b^2)}{\cancel{3b}} = 11b^2$

b) $\frac{45x^2y^3}{5xy}$ $\frac{\cancel{5xy}(9xy^2)}{\cancel{5xy}} = 9xy^2$

c) $\frac{9y-3}{15y-5}$ $\frac{3\cancel{(3y-1)}}{5\cancel{(3y-1)}} = \frac{3}{5}$

d) $\frac{k-4}{k^2-16} = \frac{\cancel{(k-4)}}{(k+4)\cancel{(k-4)}} = \frac{1}{k+4}$

2) Multiply and divide the following as indicated:

a) $\frac{5k^6}{\cancel{2}m^5} \cdot \frac{\cancel{2}m^2}{k^4} = \frac{5k^6}{k^4} \cdot \frac{m^2}{m^5} =$

$\frac{5k^2}{m^3}$

b) $\frac{\cancel{(x-2)}(x+3)}{(x+8)\cancel{(x+1)}} \cdot \frac{\cancel{(x+1)}\cancel{(x-3)}}{\cancel{(x-2)}\cancel{(x-3)}} = \frac{x+3}{x+8}$

c) $\frac{(x-7)(x-3)}{(x+2)(x-1)} \div \frac{(x+1)(x-3)}{(x+2)(x-1)} =$

$\frac{(x-7)\cancel{(x-3)}}{\cancel{(x+2)}\cancel{(x-1)}} \cdot \frac{\cancel{(x+2)}\cancel{(x-1)}}{(x+1)\cancel{(x-3)}} = \frac{x-7}{x+1}$

$D: \{x \mid x \neq 1\}$

d) $\frac{y^2-1}{3y^2-5y-2} \div \frac{y^2+y-2}{3y^2+7y+2}$

$\frac{y^2-1}{3y^2-5y-2} \cdot \frac{3y^2+7y+2}{y^2+y-2} =$

$\frac{(y+1)\cancel{(y-1)}}{\cancel{(3y+1)}(y-2)} \cdot \frac{\cancel{(3y+1)}\cancel{(y+2)}}{\cancel{(y+2)}\cancel{(y-1)}} = \frac{y+1}{y-2}$

e) $\frac{x+3}{x^7-x} \div \frac{x^2-9}{x^5}$

$\frac{\cancel{x+3}}{x(x^6-1)} \cdot \frac{\cancel{x^5}\, x^4}{(x-3)\cancel{(x+3)}} =$

$\frac{x^4}{(x^6-1)(x-3)}$

f) $\frac{3k^2+5k-12}{3k^2-k-4} \div \frac{7k^2+16k-15}{k^2+4k+3}$

$\frac{\cancel{(3k-4)}(k+3)}{\cancel{(3k-4)}\cancel{(k+1)}} \cdot \frac{\cancel{(k+3)}\cancel{(k+1)}}{(7k-5)\cancel{(k+3)}} =$

$\frac{k+3}{7k-5}$

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GPS # 26 6.3 ADDITION AND SUBTRACTION OF RATIONAL EXPRESSIONS NAME:

Useful Guidelines: [a, b, and c are nonzero real number.]
To add or subtract rational expressions:
* If the denominators are the same, add or subtract the numerators and place the result over the common denominator: $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ or $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$.

$\frac{a}{c}+\frac{b}{d}$

* If the denominators are different, then
  Step 1: Write the rational expressions with the least common denominator (LCM),
  Step 2: Add or subtract the numerators and place the result over the common denominator.
* Finally, check that the answer is in lowest terms.

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Evaluate the following:

1. a) $\frac{3}{x}-\frac{1}{2x^2} \Rightarrow \frac{2x}{2x}\cdot\frac{3}{x}-\frac{1}{2x^2}=$ $\frac{6x-1}{2x^2}$

b) $\frac{(4k+7)}{k+2}-\frac{(3k+1)}{k+2} \Rightarrow \frac{4k+7-3k-1}{k+2}=\frac{k+6}{k+2}$

(*) back

LCD= (m+2)(m-2)

2. a) $\frac{3}{(m-2)}-\frac{1}{(m+2)} \Rightarrow \frac{(m+2)}{(m+2)}\cdot\frac{3}{(m-2)}-\frac{1}{(m+2)}\frac{(m-2)}{(m-2)}$

$\frac{3(m+2)-(m-2)}{m^2-4}=\frac{3m+6-m+2}{m^2-4}=\frac{2m+8}{m^2-4}$

b) $\frac{(3a-2)}{c^2-16}-\frac{(2b+2)}{c^2-16} \Rightarrow \frac{3a-2-2b-2}{c^2-16} \Rightarrow$

$\frac{3a-2b-4}{c^2-16}$ or $\frac{3a-2b-4}{(c+4)(c-4)}$

LCD= z-25 (z-5)(z+5)

3. a) $\frac{3z}{z^2-25}+\frac{2z}{z-5}=\frac{3z}{z^2-25}+\frac{2z}{z-5}\cdot\frac{(z+5)}{(z+5)}$

$\frac{3z+2z^2+10^2}{(z+5)(z-5)}=\frac{2z^2+13z}{(z+5)(z-5)}$

LCD: (y+2)(y-2)

b) $\frac{3y}{y^2-4}+\frac{2y}{2-y}-\frac{5}{y-2}$

$\frac{3y}{(y+2)(y-2)}-\frac{2y}{(y-2)}\cdot\frac{(y+2)}{(y+2)}-\frac{5}{(y-2)}\cdot\frac{(y+2)}{(y+2)}$

$\frac{3y-2y^2-4y-5y-10}{(y+2)(y-2)}=$

$\frac{-2y^2-6y-10}{(y+2)(y-2)}$ "lowest term"

LCD=(x+1)

4. $\frac{2x}{x^2+4x+3}+\frac{x}{2x^2+3x+1}=$
(x+1)(x+3) (2x+1)(x+1)

$\frac{2x\cdot(2x+1)}{(x+1)(x+3)(2x+1)}+\frac{x(x+3)}{(2x+1)(x+1)(x+3)}=$

$\frac{4x^2+2x+x^2+3x}{(2x+1)(x+1)(x+3)}=\frac{5x^2+5x}{(2x+1)(x+1)(x+3)}=\frac{5x\cancel{(x+1)}}{(2x+1)\cancel{(x+1)}(x+3)}=\frac{5x}{(2x+1)(x+3)}$

"lowest term"

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GPS # 27 **6.4 RATIONAL EQUATIONS** **NAME:** Antoinette Durden

20/20

Useful Guidelines:

* To solve an equation with rational expressions:
  1. Determine the domain.
  2. Multiply all the terms in the equation by the least common denominator.
  3. Solve the resulting equation and check that the solution is in the domain of the equation.
* To solve a formula for a specified variable, isolate that variable on one side of the equation.
* To solve a motion problem, use the formula $d = rt$ or one of its equivalents, $t = \frac{d}{r}$ or $r = \frac{d}{t}$.
* To solve a word problem:
  (1) Assign a variable, (2) Write an equation, (3) Solve the variable, and (4) State the answer.

1. Solve each equation: multiply Both sides by LCD 3x

a) $\frac{1}{x} - \frac{4}{3} = \frac{2}{3x} =$

LCD= 3x
D: $\{x \mid x \neq 0\}$

$3x\left(\frac{1}{x} - \frac{4}{3}\right) = \frac{2}{3x}(3x)$

$3x\left(\frac{1}{x}\right) - 3x\left(\frac{4}{3}\right) = 2$

$3 - 4x = 2 =$

$\frac{-4x}{-4} = \frac{-1}{-4}$

$x = \frac{1}{4}$

Sol set $\{x \mid x = \frac{1}{4}\}$

b) $\frac{3}{x-2} - \frac{1}{x+2} = \frac{8}{x^2-4} =$

$(x+2)(x-2)$

LCD= x^2-4
D: $\{x \mid x \neq 2$ or $x \neq -2\}$

$3(x+2) - (x-2) = 8$

$3x + 6 - x + 2 = 8$

$2x + 8 = 8$

$2x = 0$

$x = 0$

Sol set: $\{x \mid x = 0\}$

2. Solve each formula for the specified variable.

LCD= d^2

a) Solve $F = \frac{GMm}{d^2}$ for m.

$\frac{d^2F}{GM} = \frac{GMm}{GM} \Rightarrow \frac{d^2F}{GM} = m$

b) Solve $\frac{PV}{T} = \frac{pv}{t}$ for V.

$\frac{PVt}{Pt} = \frac{pvT}{Pt} \Rightarrow V = \frac{pvT}{Pt}$

3. If 20 out of 100 Americans had no dental insurance coverage. The population at that time was about 280 million. How many million had no dental insurance coverage? [Hint: Let $x =$ the number (in millions) who had no dental insurance.]

LCD=1400

x can cross multiply

$\frac{20}{100} = \frac{x}{280} \Rightarrow 1400\left(\frac{20}{100}\right) = \left(\frac{x}{280}\right)1400 = \frac{5x}{5} = \frac{4(14)}{5} =$

$x = 56$ million

4. Jonathan's car uses 5 gallon of gas to travel 100 miles. He has 4 gallon of gas in the car, and he wants to know how much more gas he will need to drive 320 miles. If we assume the car continues to use gas at the same rate, how many more gallons will he need? [Hint: Let $x =$ the additional number of gallon of gas needed.]

5 gallons — 100 miles
(x+4) gall — 320 miles

$\frac{x+4 \text{ gallons}}{320 \text{ miles}} = \frac{5 \text{ gallons}}{100 \text{ miles}}$

$\frac{(x+4) \text{ gal}}{320 \text{ miles}} = \frac{1 \text{ gallons}}{20 \text{ miles}}$

$320\left(\frac{x+4}{320}\right) = \left(\frac{1}{20}\right)320$

$x + 4 = 16$

$x = 12$ gallons

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GPS # 28 **6.5 COMPLEX FRACTIONS** **NAME:** Antoinette Durden

20/20

Useful Guidelines:
To simplify complex fractions:
* Step 1: Identify the least common denominator for both the fractions in the numerator and denominator.
* Step 2: Multiply the numerator and denominator by the least common denominator.
* Step 3: Simplify the result. [Hint: If possible, factor out the common factor.]

Simplify the following complex fractions:

1. a) $\dfrac{\left(-\frac{5}{\cancel{3}}\right)\cancel{18}\,6}{\left(\frac{1}{\cancel{18}}\right)\cancel{18}}$ $\dfrac{-30}{1} = -30$

LCD: 18

LCD: 8

b) $\dfrac{\left(\frac{1}{4}+\frac{5}{2}\right)8}{\left(1-\frac{1}{8}\right)8} = \dfrac{2+20}{8-1} = \dfrac{22}{7}$

LCD: 6x

2. a) $\dfrac{\left(\frac{x+1}{\cancel{3x}}\right)\cancel{6x}\,2}{\left(\frac{3x-1}{6x}\right)6x} = \dfrac{2x+2}{3x-1}$

LCD: y^2

b) $\dfrac{\left(\frac{1}{y}+\frac{25}{y^2}\right)y^2}{\left(\frac{7}{y}-\frac{1}{y^2}\right)y^2} = \dfrac{y+25}{7y-1}$

LCD: a^2b^2

3. a) $\dfrac{\left(\frac{20}{a^2b^2}+\frac{3}{ab^2}\right)a^2b^2}{\left(\frac{9}{a^2b}-\frac{4}{a^2b^2}\right)a^2b^2} = \dfrac{20+3c}{9b-4}$

b) $\dfrac{4}{x-3y} \div \dfrac{2}{x^2-9y^2} = \dfrac{\frac{4}{x-3y}}{\frac{2}{x^2-9y^2}} = \dfrac{\left(\frac{4}{\cancel{x-3y}}\right)}{\frac{2}{\cancel{(x+3y)}\cancel{(x-3y)}}}$

$(x+3y)\cancel{(x-3y)}$

$\cancel{(x+3y)}\cancel{(x-3y)}$

multiply $= \dfrac{\overset{2}{\cancel{4}}(x+3y)}{\cancel{2}} = 2x+6y$

$\cancel{(x+2)}\cancel{(x-5)}$

4. $\dfrac{\left(\frac{3}{\cancel{x+2}}-\frac{1}{\cancel{x-5}}\right)}{\left(\frac{9}{\cancel{x-5}}+\frac{1}{\cancel{x+2}}\right)}$ $\dfrac{3(x-5)-(x+2)}{9(x+2)+(x-5)} =$

$\cancel{(x+2)}\cancel{(x-5)}$

$\dfrac{(3x-15)-(x-2)}{(9x+18)+(x-5)} = \dfrac{2x-17}{10x+13}$

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GPS # 29 **6.6 MODELING WITH PROPORTIONS AND VARIATIONS** **NAME:** Antoinette Durden

Useful Definitions: [k is a constant.]

* Proportion: A statement that two ratios are equal. Example: $\frac{x}{2}=\frac{3}{4}$

* $y = kx$: y varies directly as x. Example: $V=\frac{4}{3}\pi r^3$ (volume of a sphere) increase

* $y=\frac{k}{x}$: y varies inversely as x. Example: $\downarrow r=\frac{d}{t\uparrow}$ (speed of a vehicle) decrease

* $y = kxz$: y varies jointly as x and z. Example: $I = prt$ (simple interest of an investment)

1. Find the missing number in the proportion.

a) $\left(\frac{x}{10}\right)=\left(\frac{7}{5}\right)$ $x = 14$

b) $\left(\frac{k}{2}\right)=\left(\frac{4}{3}\right)$ $3k=8$ $k=\frac{8}{3}$

c) $\left(\frac{3}{4}\right)=\left(\frac{x}{16}\right)$ $12 = x$

d) $\left(\frac{y}{30}\right)=\left(\frac{30}{45}\right)$ $3y = 60$ $y = 20$

2. The voltage (y) in a simple electrical circuit is directly proportional to the resistance (x). If the voltage is 5 volts when the resistance is 15 ohms, find the voltage when the resistance is 24 ohms.

$\frac{(5)}{15}=\frac{k(15)}{15}$ Find y when $x = 24$

$\frac{1}{3}=k \Rightarrow y=\frac{1}{3}x$ $y=\frac{1}{3}(24) = 8$ volts

3. Body mass index, or BMI, is used by physicians to access a person's level of fatness. A BMI from 19 through 25 is considered desirable. BMI varies directly as an individual's weight in pounds and inversely as the square of the individual's height in inches. A person who weights 120 lbs and is 60 in. tall has a BMI of 20. (Source: Washington Post.) Find the BMI of a person who weights 150 lb with a height of 60 in.

Let weight $= x$
height $= y$

BMI: $\frac{kx}{y^2}$ $20=\frac{k(120)}{(60)(60)}$ $20=\frac{k}{30}$ $k = 600$

$BMI = \frac{600x}{y^2} = \frac{(600)(150)}{(60)(60)}$ $\frac{150}{6} = 25$

4. As simple interest varies jointly as principal and time, if your investment of $1,000, left in a mutual fund for 2 years, earned you an interest of $200. How much interest would you expect to earn in 10 years?

$I = kpt$

$200 = k(1000)(2)$

$\frac{200}{2000}=\frac{2000k}{2000}$

$\frac{1}{10}=k$

$I=\frac{1}{10}pt$

$=\frac{1}{10}(1000)(10) = \$1,000$