INTERMEDIATE ALGEBRA

GPS # 32 **7.3 OPERATIONS ON RADICAL EXPRESSIONS I** **NAME:** Antoinette Durden

20/20 Good Job!

Useful Guidelines:

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and $b \neq 0$ and n is a natural number, then

* $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ and $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$. For example: $\sqrt[4]{8} \cdot \sqrt[4]{2} = \sqrt[4]{8 \cdot 2} = 2$ and $\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$

* NOTE: $\sqrt[n]{a} + \sqrt[n]{b} \neq \sqrt[n]{a+b}$ and $\sqrt[n]{a} - \sqrt[n]{b} \neq \sqrt[n]{a-b}$

Only radical expression with the same index and the same radicand may be combined (combining the like terms or same family only).

For example: $\sqrt[3]{5} + 2\sqrt[3]{5} + 4\sqrt[3]{7} + 2\sqrt[3]{7} = 3\sqrt[3]{5} + 6\sqrt[3]{7}$.

Assume all variables represent positive real numbers. Simplify the following (if possible):

1. a) $\sqrt[3]{15} + 6\sqrt[3]{15} = 7\sqrt[3]{15}$

b) $\sqrt{20} - \sqrt{45} = \sqrt{(4)(5)} - \sqrt{(9)(5)}$
$= 2\sqrt{5} - 3\sqrt{5}$
$= -\sqrt{5}$

c) $2\sqrt{45a} + \sqrt{20a}$
$= 2\sqrt{(9)(5)(a)} + \sqrt{(4)(5)(a)}$
$= 6\sqrt{5a} + 2\sqrt{5a}$
$= 8\sqrt{5a}$

d) $3\sqrt{5} - 5\sqrt{2}$

Cannot simplify

2. a) $3\sqrt[3]{16} - 2\sqrt[3]{250}$
$= 3\sqrt[3]{(8)(2)} - 2\sqrt[3]{(125)(2)}$
$= 6\sqrt[3]{2} - 10\sqrt[3]{2}$
$= -4\sqrt[3]{2}$

b) $5\sqrt{12} - 3\sqrt{27} = 5\sqrt{(4)(3)} - 3\sqrt{(9)(3)}$
$= 10\sqrt{3} - 9\sqrt{3}$
$= \sqrt{3}$

c) $\sqrt[4]{81x^2y} + \sqrt[4]{16x^6y^5}$
$= 3\sqrt[4]{x^2y} + \sqrt[4]{(2^4)(x^2x^4)(y^4y^1)}$
$= 3\sqrt[4]{x^2y} + 2xy\sqrt[4]{x^2y}$
$= (3+2xy)\sqrt[4]{x^2y}$

d) $\sqrt[3]{8m^2n} + 3\sqrt[3]{m^5n^7}$
$= 2\sqrt[3]{m^2n} + 3\sqrt[3]{(m^3m^2)(n^3n^3n)}$
$= 2\sqrt[3]{m^2n} + 3mn^2\sqrt[3]{m^2n}$
$= 2+3mn^2\sqrt[3]{m^2n}$

3. a) $2\sqrt{\frac{32}{25}} + 3\sqrt{\frac{32}{8}} = 2\frac{\sqrt{32}}{\sqrt{25}} + 3\frac{\sqrt{32}}{\sqrt{8}} = \frac{2\sqrt{(16)(2)}}{5} + \frac{3\sqrt{(16)(2)}}{\sqrt{(4)(2)}} =$
$\frac{8\sqrt{2}}{5} + \frac{\cancel{12\sqrt{2}}^{6}}{\cancel{2\sqrt{2}}} = \frac{8\sqrt{2}}{5} + 6$

c) $\sqrt[3]{\frac{8x^3}{x^{12}}} + \sqrt[3]{\frac{16}{x^9}} = \frac{\sqrt[3]{8x^3}}{\sqrt[3]{x^{12}}} + \frac{\sqrt[3]{16}}{\sqrt[3]{x^9}} = \frac{2x}{x^4} + \frac{\sqrt[3]{(8)(2)}}{x^3} = \frac{2x}{x^4} + \frac{2\sqrt[3]{2}}{x^3} =$
$\frac{2}{x^3} + \frac{2\sqrt[3]{2}}{x^3} = \frac{2+2\sqrt[3]{2}}{x^3}$

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GPS # 33 7.3 OPERATIONS ON RADICAL EXPRESSIONS II NAME: Antoinette Duden

Useful Guidelines:

* To rationalize denominator with one radical term:
Multiply both the numerator and the denominator by that radical term in the denominator.

For example: $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

* To rationalize denominator with binomials involving radicals:
Multiply both the numerator and the denominator by the conjugate of the denominator.

For example: $\frac{3}{4+\sqrt{5}} = \frac{3}{(4+\sqrt{5})} \cdot \frac{(4-\sqrt{5})}{(4-\sqrt{5})} = \frac{3(4-\sqrt{5})}{16-5} = \frac{3(4-\sqrt{5})}{11}$

Rationalize the denominator in each expression. Assume all variables represent positive real numbers.

1. a) $\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$

b) $\frac{12}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{12\sqrt{7}}{7}$

c) $\sqrt{\frac{3}{11}} = \frac{\sqrt{3}}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{\sqrt{33}}{11}$

d) $-\sqrt{\frac{13}{x}} = -\frac{\sqrt{13}}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = -\frac{\sqrt{13x}}{x}$

e) $\sqrt[3]{\frac{27}{2}} = \frac{3}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}\,\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{3\sqrt[3]{4}}{2}$

f) $-\sqrt[4]{\frac{16}{y}} = \frac{-\sqrt[4]{16}}{\sqrt[4]{y}} \frac{2\sqrt[4]{y}\sqrt[4]{y}\sqrt[4]{y}\sqrt[4]{y}}{\sqrt[4]{y}\sqrt[4]{y}\sqrt[4]{y}} = \frac{-2\sqrt[4]{y^3}}{y}$

Rationalize the denominator in each expression. Assume all variables represent positive real numbers and no denominators are 0.

2. a) $\frac{8}{(3-\sqrt{5})} \frac{(3+\sqrt{5})}{(3+\sqrt{5})} = \frac{24+8\sqrt{5}}{9-5} = \frac{\overset{6}{24}+\overset{2}{8}\sqrt{5}}{\underset{1}{4}} = 6+2\sqrt{5}$

b) $\frac{4}{(\sqrt{3}-\sqrt{2k})} \cdot \frac{(\sqrt{3}+\sqrt{2k})}{(\sqrt{3}+\sqrt{2k})} = \frac{4\sqrt{3}+4\sqrt{2k}}{3-2k}$

c) $\frac{(\sqrt{2}-\sqrt{3})}{(\sqrt{4}+\sqrt{3})} \frac{(\sqrt{4}-\sqrt{3})}{(\sqrt{4}-\sqrt{3})} = \frac{2\sqrt{2}-2\sqrt{3}-\sqrt{6}+3}{4-3} =$

$2\sqrt{2}-2\sqrt{3}-\sqrt{6}+3$

d) $\frac{(3+\sqrt{2x})}{(\sqrt{3}-\sqrt{2x})} \frac{(\sqrt{3}+\sqrt{2x})}{(\sqrt{3}+\sqrt{2x})} = \frac{3\sqrt{3}+\sqrt{6x}+3\sqrt{2x}+2x}{3-2x}$

$\frac{3\sqrt{3}+\sqrt{6x}+3\sqrt{2x}+2x}{3-2x}$

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GPS # 34 7.4 RADICAL FUNCTIONS NAME: Antinette Durden

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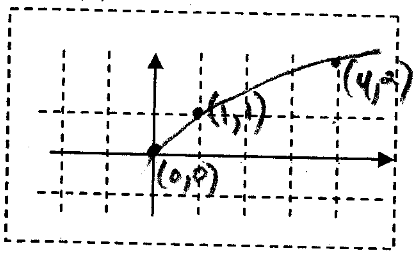
Useful Guidelines:

* Square Root and Cube Root: The cube root function is defined for all inputs, whereas the square root function is defined only for nonnegative inputs.
* Square Root Property: If k is a nonnegative numbers and $x^2 = k$, then $x = \pm\sqrt{k}$.
* Solve Equations with Cube Roots: The solution to the equation $x^3 = k$ is $x = \sqrt[3]{k}$.
* Power function: $f(x) = x^p$, where p is a rational number.
* Root function: $f(x) = \sqrt[n]{x}$, where $n \geq 2$.

1. Graph each function. Give the domain and the range.

a. $f(x) = \sqrt{x}$

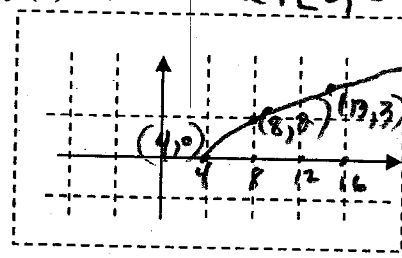


x	$f(x)$
0	0
1	1
4	2

D: $[0, \infty)$
R: $[0, \infty)$

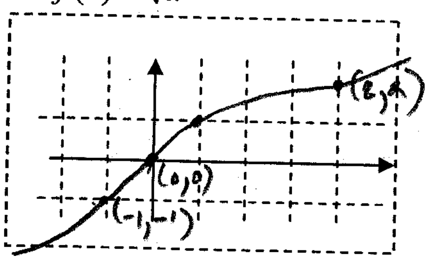
b. $f(x) = \sqrt{x-4}$

D: $[4, \infty)$
R: $[0, \infty)$



x	$f(x)$
4	0
8	2
13	3

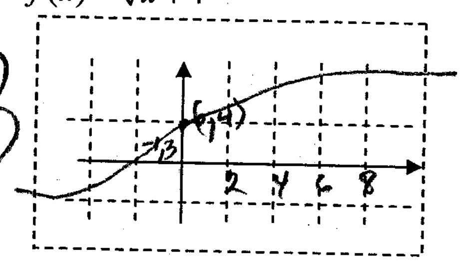
c. $f(x) = \sqrt[3]{x}$



x	$f(x)$
0	0
1	1
8	2
27	3
-1	-1

D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

d. $f(x) = \sqrt[3]{x} + 4$



x	$f(x)$
0	4
1	5
8	6

D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

2. Use the square root property to solve each equation and give the solution set.

a) $x^2 = 49$

$x = \pm\sqrt{49}$

$x = \pm 7$

Sol set $\{x \mid x = \pm 7\}$

b) $x^2 - 8 = 0$ $x = 8$

$x = \pm\sqrt{8}$

$x = \pm 2\sqrt{2}$

Sol set: $\{x \mid x = \pm 2\sqrt{2}\}$

c) $(x-4)^2 = 25$

$x - 4 = \pm 5$

$x - 4 = 5$ or $x - 4 = -5$

$x = 9$ or $x = -1$

Sol set $\{x \mid x = 9 \text{ or } x = -1\}$

d) $(2x-5)^2 = 12$

$2x - 5 = \pm 2\sqrt{3}$

$2x - 5 = 2\sqrt{3}$ or $2x - 5 = -2\sqrt{3}$

$2x = 5 + 2\sqrt{3}$ $2x = 5 - 2\sqrt{3}$

$x = \dfrac{5 + 2\sqrt{3}}{2}$ $x = \dfrac{5 - 2\sqrt{3}}{2}$

3. Use the cube roots to solve each equation and give the solution set.

a) $x^3 = 125$

$x = \sqrt[3]{125}$

$x = 5$

Sol set $\{5\}$

b) $\dfrac{3(y-1)^3}{3} = \dfrac{81}{3} \Rightarrow (y-1)^3 = 27$

$y - 1 = \sqrt[3]{27}$

$y - 1 = 3$

$y = 4$ Sol set $\{4\}$

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GPS # 36 7.6 COMPLEX NUMBERS NAME: Antoinette Durden

Useful Guidelines:

* Imaginary Unit: $i=\sqrt{-1}$ and $i^2=-1$. Example: $\sqrt{-5}=i\sqrt{5}$.
* Standard Form: $a+bi$, where a and b are real numbers.
* Real Part of $a+bi$: a
* Imaginary Part of $a+bi$: b
* Complex conjugate of $a+bi$: $a-bi$

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1. Use the imaginary unit to write the expression.

a. $\sqrt{-36}$ $\sqrt{(-1)(36)} \Rightarrow \sqrt{-1}\sqrt{36} \Rightarrow 6i$

b. $\sqrt{-81}$ $\sqrt{(-1)(81)} \Rightarrow \sqrt{-1}\sqrt{81} = 9i$

c. $\sqrt{-72}$ $\sqrt{(-1)(36)(2)} = \sqrt{-1}\sqrt{36}\sqrt{2} = 6i\sqrt{2}$

d. $\sqrt{-20} = \sqrt{(-1)(5)(4)} = 2i\sqrt{5}$

2. Write each sum or difference in standard form.

a. $(2+i)+(9-2i)$ $2+i+9-2i = 11-i$

b. $(4-5i)-(1-i)$ $4-5i-1+i = 3-4i$

c. $(-3+2i)+(4-4i)$ $-3+2i+4-4i = 1-2i$

d. $(6-5i)-(2+5i)$ $6-5i-2-5i = 4-10i$

3. Write each product in standard form.

$i^2=-1$

a. $(3-i)(2+2i)$

$6-2i+6i-2(-1)$

$8+4i$

b. $(7+i)(5-2i)$

$35+5i-14i-2(-1)$

$37-9i$

c. $(-4+3i)(2+8i)$

$-8+6i-32i+24\overset{(-1)}{i^2}$

$-32-26i$

d. $(5-4i)(1-3i)$

$5-4i-15i+12\overset{(-1)}{i^2}$

$-7-19i$

4. Write each quotient in standard form.

a. $\dfrac{3}{4-3i}\dfrac{(4+3i)}{(4+3i)} \Rightarrow \dfrac{12+9i}{16-9i^2} =$

$\dfrac{12+9i}{16-9(-1)} \Rightarrow \dfrac{12+9i}{25} = \dfrac{12}{25}+\dfrac{9}{25}i$

b. $\dfrac{2i}{2+5i}\dfrac{(2-5i)}{(2-5i)} \Rightarrow \dfrac{4i-10i^2}{4-25i^2} =$

$\dfrac{4i-10(-1)}{4-25(-1)} \Rightarrow \dfrac{4i+10}{29} =$

$\dfrac{10}{29}+\dfrac{4}{29}i$

c. $\dfrac{2-i}{3+4i}\dfrac{(3-4i)}{(3-4i)} \Rightarrow \dfrac{6-3i-8i+4i^2}{9-16i^2} =$

$\dfrac{6-11i+4(-1)}{9-16(-1)} = \dfrac{2-11i}{25} =$

$\dfrac{2}{25}-\dfrac{11}{25}i$

d. $\dfrac{1+2i}{5i}\dfrac{(-5i)}{(-5i)} \Rightarrow \dfrac{-5i-10i^2}{-25i^2} =$

$\dfrac{-5i-10(-1)}{(-25)(-1)} = \dfrac{-5i+10}{25}$

$-\dfrac{\cancel{5}^{1}}{\cancel{25}_{5}}i+\dfrac{\cancel{10}^{2}}{\cancel{25}_{5}} = \dfrac{2}{5}-\dfrac{1}{5}i$