

INTERMEDIATE ALGEBRA

GPS # 30

7.1 RADICAL EXPRESSIONS AND RATIONAL EXPONENTS

NAME: Brou Patel

Useful Guidelines:

- * $\sqrt[n]{a} = a^{\frac{1}{n}} = b$ means $a = b^n$, where $\sqrt[n]{a}$ is the principal n^{th} root of a .
- * $\sqrt[n]{a^n} = |a|$ if n is even. For example: $\sqrt[4]{(-3)^4} = |(-3)| = 3$.
- * $\sqrt[n]{a^n} = a$ if n is odd. For example: $\sqrt[3]{(-5)^3} = -5$.

My Good No Calc!

[Remember: $(a^n)^m = a^{nm}$, $a^m a^n = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$]

Rational Exponents:

- * If m and n are positive integers with $\frac{m}{n}$ in lowest term, then $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$.

[Remember: $(ab)^m = a^m b^m$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$]

Simplify each root so that no radicals appear. Assume all variables represents real numbers.

1. a) $\sqrt{81} = (9^2)^{\frac{1}{2}} = 9$

b) $\sqrt[3]{8} = (2^3)^{\frac{1}{3}} = 2$

c) $\sqrt[3]{32} = (2^5)^{\frac{1}{3}} = 2$

even roots of ± are absolute value
d) $\sqrt[4]{(-3)^4} = (-3^4)^{\frac{1}{4}} = |-3| = 3$

e) $-\sqrt[3]{(-5)^3} = -(-5^3)^{\frac{1}{3}} = 5$

f) $-\sqrt[4]{(-2)^4} = -(-2^4)^{\frac{1}{4}} = -2$

2. a) $\sqrt{5^2} = (5^2)^{\frac{1}{2}} = 5$

b) $\sqrt{(-5)^2} = |-5| = 5$

c) $\sqrt[4]{(-5)^4} = |-5| = 5$

d) $\sqrt{x^2} = |x|$

e) $\sqrt[3]{x^9} = (x^9)^{\frac{1}{3}} = x^3$

f) $-\sqrt[3]{27t^3} = -(3t)^3^{\frac{1}{3}} = -3t$

g) $\sqrt[3]{32k^5} = (2k)^5^{\frac{1}{3}} = 2k$

h) $-\sqrt{x^2} = -|x|$ *absolute value*

Evaluate the following, simplify if possible, and write the answer with only positive exponents.

3. a) $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^2 = 4$

b) $(16)^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^3 = 8$

flip to make
d) $-\left(\frac{16}{81}\right)^{\frac{3}{4}} = -\left(\frac{2^4}{3^4}\right)^{\frac{3}{4}} = -\frac{2^3}{3^3} = -\frac{8}{27}$

4. a) $3^{\frac{1}{3}} \cdot 3^{\frac{5}{3}} = 3^{\frac{1}{3} + \frac{5}{3}} = 3^2 = 9$

b) $x^{-\frac{1}{2}}(x^{\frac{5}{2}} + x) = x^{-\frac{1}{2} + \frac{5}{2}} + x^{-\frac{1}{2} + 1} = x^2 + x^{\frac{1}{2}} = x^2 + \sqrt{x}$

5. a) $\frac{t^{\frac{5}{6}} \cdot t^{\frac{2}{6}}}{t} = t^{\frac{5}{6} + \frac{2}{6} - 1} = t^{\frac{5+2-6}{6}} = t^{\frac{1}{6}} = \sqrt[6]{t}$

xb) $\sqrt[3]{x^4} \cdot \sqrt[3]{x^6} = x^{\frac{4}{3}} \cdot x^2 = x^{\frac{4}{3} + 2} = x^{\frac{10}{3}}$

OR $\sqrt[3]{x^4 \cdot x^6} = \sqrt[3]{x^{10}}$

~~Roots~~

<http://faculty.valenciac.edu/ashaw/>

INTERMEDIATE ALGEBRA

GPS # 31

7.2 SIMPLIFYING RADICAL EXPRESSIONS

NAME:

Parul Patel

Useful Guidelines:

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and $b \neq 0$ and n is a natural number, then

* $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ and $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$. For example: $\sqrt[4]{8} \cdot \sqrt[4]{2} = \sqrt[4]{8 \cdot 2} = 2$ and $\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$ *no*

* Rationalizing the Denominator:

Writing the quotient without a radical expression in the denominator.

Evaluate the following:

1. a) $\sqrt[3]{9} \cdot \sqrt[3]{3} = \sqrt[3]{9 \cdot 3} = \sqrt[3]{27} = 3$

b) $\sqrt[3]{16} \cdot \sqrt[3]{4} = \sqrt[3]{16 \cdot 4} = \sqrt[3]{64} = 4$

c) $\sqrt[5]{9t} \cdot \sqrt[5]{5} = \sqrt[5]{45t}$

d) $\sqrt{4m} \cdot \sqrt{3pq} = \sqrt{12mpq}$

e) $\sqrt[4]{10} \cdot \sqrt[4]{5} = \sqrt[4]{50}$

f) $\sqrt[5]{4x} \cdot \sqrt[3]{y} = \text{Can't Simplify}$

Assume all variables represent positive real numbers. Simplify the following (if possible):

2. a) $\sqrt[3]{-\frac{64}{8}} = \left(\frac{-4}{2}\right)^{\frac{3}{3}} = -\frac{4}{2} = -2$

b) $\sqrt{\frac{81}{49}} = \frac{9}{7}$

c) $\sqrt[4]{\frac{81}{16}} = \left(\frac{3}{2}\right)^{\frac{4}{4}} = \frac{3}{2}$

d) $\sqrt[3]{\frac{k^9}{125}} = \left(\frac{k^3}{5}\right)^{\frac{3}{3}} = \frac{k^3}{5}$

e) $\sqrt{\frac{64m^4}{a^2}} = \frac{\sqrt{64m^4}}{\sqrt{a^2}} = \frac{8m^2}{a}$

f) $\sqrt[4]{\frac{x^4}{16y^8z^2}} = \frac{x}{2y^2z^{\frac{1}{2}}}$

3. a) $\sqrt[3]{54} = \sqrt[3]{(2)(27)} = \sqrt[3]{2} \cdot \sqrt[3]{27} = 3\sqrt[3]{2}$

b) $-\sqrt[3]{32} = -\sqrt[3]{(8)(4)} = -\sqrt[3]{8} \cdot \sqrt[3]{4} = -2\sqrt[3]{4}$

c) $-\sqrt{11} = \text{Can't Simplify}$

d) $\sqrt{4x^3} = \sqrt{4 \cdot x^3} = \sqrt{4} \cdot \sqrt{x^3} = 2\sqrt{x^3}$

e) $\sqrt[3]{27p^5q^4} = \sqrt[3]{27} \sqrt[3]{p^3q^3} \sqrt[3]{pq} = 3pq\sqrt[3]{pq}$

f) $-\sqrt[4]{32a^5} = -\sqrt[4]{16 \cdot 2 \cdot 4 \cdot a^4} = -2a\sqrt[4]{2a}$

4. Rationalize each denominator. Assume that all variables are positive.

a) $\frac{7}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{3}$

b) $\frac{5}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{5\sqrt{2}}{4} = \frac{5\sqrt{2}}{4}$

c) $\frac{7\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{7\sqrt{15}}{5}$

d) $\frac{4\sqrt{a}\sqrt{b}}{\sqrt{b}\sqrt{b}} = \frac{4\sqrt{ab}}{b}$

e) $\sqrt{\frac{9m^2}{n}} = \frac{\sqrt{9m^2}}{\sqrt{n}} = \frac{3m}{\sqrt{n}} \cdot \frac{\sqrt{n}}{\sqrt{n}} = \frac{3m\sqrt{n}}{n}$

f) $\frac{2xy}{\sqrt{x^3}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{2xy\sqrt{x}}{x\sqrt{x}} = \frac{2y\sqrt{x}}{\sqrt{x}}$

INTERMEDIATE ALGEBRA

GPS # 32

7.3 OPERATIONS ON RADICAL EXPRESSIONS I

NAME: Parul Patel

Useful Guidelines:

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and $b \neq 0$ and n is a natural number, then

* $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ and $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$. For example: $\sqrt[4]{8} \cdot \sqrt[4]{2} = \sqrt[4]{8 \cdot 2} = 2$ and $\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$

* NOTE: $\sqrt[n]{a} + \sqrt[n]{b} \neq \sqrt[n]{a+b}$ and $\sqrt[n]{a} - \sqrt[n]{b} \neq \sqrt[n]{a-b}$

Only radical expression with the same index and the same radicand may be combined (combining the like terms or same family only).

For example: $\sqrt[3]{5} + 2\sqrt[3]{5} + 4\sqrt[3]{7} + 2\sqrt[3]{7} = 3\sqrt[3]{5} + 6\sqrt[3]{7}$.

Assume all variables represent positive real numbers. Simplify the following (if possible):

1. a) $\sqrt[3]{15} + 6\sqrt[3]{15}$

$$= 7\sqrt[3]{15}$$

b) $\sqrt{20} - \sqrt{45}$

$$= 2\sqrt{5} - 3\sqrt{5}$$

$$= -\sqrt{5}$$

c) $2\sqrt{45a} + \sqrt{20a}$

$$= 2\sqrt{9 \cdot 5a} + \sqrt{4 \cdot 5a}$$

$$= 2 \cdot 3\sqrt{5a} + 2\sqrt{5a}$$

$$= 6\sqrt{5a} + 2\sqrt{5a} = 8\sqrt{5a}$$

d) $3\sqrt{5} - 5\sqrt{2}$

can't simplify

2. a) $3\sqrt[3]{16} - 2\sqrt[3]{250}$

$$= 3 \cdot \sqrt[3]{8 \cdot 2} - 2 \cdot \sqrt[3]{125 \cdot 2}$$

$$= 3 \cdot 2\sqrt[3]{2} - 2 \cdot 5\sqrt[3]{2}$$

$$= 6\sqrt[3]{2} - 10\sqrt[3]{2}$$

$$= -4\sqrt[3]{2}$$

b) $5\sqrt{12} - 3\sqrt{27}$

$$= 5\sqrt{4 \cdot 3} - 3\sqrt{9 \cdot 3}$$

$$= 5 \cdot 2\sqrt{3} - 3 \cdot 3\sqrt{3}$$

$$= 10\sqrt{3} - 9\sqrt{3}$$

$$= \sqrt{3}$$

c) $\sqrt[4]{81x^2y} + \sqrt[4]{16x^6y^5}$

$$= 3\sqrt[4]{x^2y} + 2xy\sqrt[4]{x^2y}$$

$$= (3 + 2xy)\sqrt[4]{x^2y}$$

d) $\sqrt[3]{8m^2n} + 3\sqrt[3]{m^5n^7}$

$$= 2\sqrt[3]{m^2n} + 3mn^2\sqrt[3]{m^2n}$$

$$= (2 + 3mn^2)\sqrt[3]{m^2n}$$

3. a) $2\sqrt{\frac{32}{25}} + 3\sqrt{\frac{32u}{8}}$

$$= 2\frac{\sqrt{32}}{\sqrt{25}} + 3 \cdot 2 = 2 \cdot \frac{\sqrt{16 \cdot 2}}{5} + 6 = \frac{8\sqrt{2}}{5} + 6 = \frac{8\sqrt{2} + 30}{5}$$

c) $\sqrt[3]{\frac{8x^3}{x^{12}}} + \sqrt[3]{\frac{16}{x^9}}$

$$= \frac{2x}{x^4} + \frac{\sqrt[3]{8 \cdot 2}}{\sqrt[3]{x^9}} = \frac{2}{x^3} + \frac{2\sqrt[3]{2}}{x^3} = \frac{2 + 2\sqrt[3]{2}}{x^3}$$

can't add
2+2 diff
different
object.

INTERMEDIATE ALGEBRA

GPS # 33

7.3 OPERATIONS ON RADICAL EXPRESSIONS II

NAME: Pamul Patel

Useful Guidelines:

* To rationalize denominator with one radical term:

Multiply both the numerator and the denominator by that radical term in the denominator.

For example: $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

* To rationalize denominator with binomials involving radicals:

Multiply both the numerator and the denominator by the conjugate of the denominator.

For example: $\frac{3}{4+\sqrt{5}} = \frac{3}{(4+\sqrt{5})} \cdot \frac{(4-\sqrt{5})}{(4-\sqrt{5})} = \frac{3(4-\sqrt{5})}{16-5} = \frac{3(4-\sqrt{5})}{11}$

Rationalize the denominator in each expression. Assume all variables represent positive real numbers.

1. a) $\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

b) $\frac{12}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{12\sqrt{7}}{7}$

c) $\sqrt{\frac{3}{11}} = \frac{\sqrt{3}}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{\sqrt{33}}{11}$

d) $-\sqrt{\frac{13}{x}} = -\frac{\sqrt{13}}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = -\frac{\sqrt{13x}}{x}$

to get 25 ans.
 $\sqrt[3]{\frac{27}{2}} = \frac{\sqrt[3]{27}}{\sqrt[3]{2}} = \frac{3}{\sqrt[3]{2}}$
 $= \frac{3}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{3\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{3\sqrt[3]{4}}{2}$

e) $\sqrt[3]{\frac{27}{2}} = \frac{3}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{3\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{3\sqrt[3]{4}}{2}$

f) $-\sqrt[4]{\frac{16}{y}} = -\frac{\sqrt[4]{16}}{\sqrt[4]{y}} = -\frac{2}{\sqrt[4]{y}}$
 So, $-\frac{2}{\sqrt[4]{y}} \cdot \frac{\sqrt[4]{y^3}}{\sqrt[4]{y^3}} = \frac{-2\sqrt[4]{y^3}}{y}$

Rationalize the denominator in each expression. Assume all variables represent positive real numbers and no denominators are 0.

Use diff of squares
 $(x-y)(x+y) = x^2 - y^2$
 multiply by conjugate

2. a) $\frac{8}{3-\sqrt{5}} \cdot \frac{(3+\sqrt{5})}{(3+\sqrt{5})} = \frac{24+8\sqrt{5}}{3^2-\sqrt{5}^2} = \frac{24+8\sqrt{5}}{9-5} = \frac{24+8\sqrt{5}}{4} = 6+2\sqrt{5}$

b) $\frac{4}{(\sqrt{3}-\sqrt{2k})} \cdot \frac{(\sqrt{3}+\sqrt{2k})}{(\sqrt{3}+\sqrt{2k})} = \frac{4\sqrt{3}+4\sqrt{2k}}{3-2k}$

c) $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{4}+\sqrt{3}} \cdot \frac{\sqrt{4}+\sqrt{3}}{\sqrt{4}-\sqrt{3}} = \frac{2\sqrt{2}-2\sqrt{3}-\sqrt{6}+3}{4-3(1)} = 2\sqrt{2}-2\sqrt{3}-\sqrt{6}+3$

d) $\frac{3+\sqrt{2x}}{\sqrt{3}-\sqrt{2x}} \cdot \frac{(\sqrt{3}+\sqrt{2x})}{(\sqrt{3}+\sqrt{2x})} = \frac{3\sqrt{3}+\sqrt{6x}+\sqrt{6x}+2x}{3-2x}$

INTERMEDIATE ALGEBRA

GPS # 34

7.4 RADICAL FUNCTIONS

NAME:

Pooal Patel

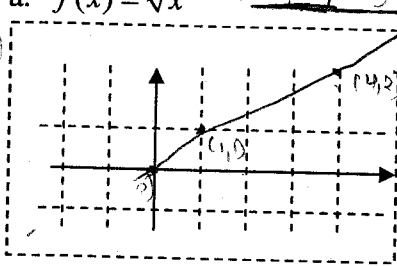
Useful Guidelines:

- * Square Root and Cube Root: The cube root function is defined for all inputs, whereas the square root function is defined only for nonnegative inputs.
- * Square Root Property: If k is a nonnegative numbers and $x^2 = k$, then $x = \pm\sqrt{k}$.
- * Solve Equations with Cube Roots: The solution to the equation $x^3 = k$ is $x = \sqrt[3]{k}$.
- * Power function: $f(x) = x^p$, where p is a rational number.
- * Root function: $f(x) = \sqrt[n]{x}$, where $n \geq 2$.

1. Graph each function. Give the domain and the range.

a. $f(x) = \sqrt{x}$ D: $[0, \infty)$ R: $[0, \infty)$

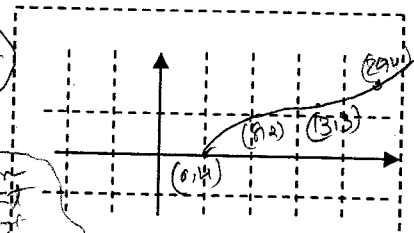
x	y
0	0
1	1
4	2
9	3



moves side ways when constant in side of Root

b. $f(x) = \sqrt{x-4}$ D: $[4, \infty)$ R: $[0, \infty)$

D: $[4, \infty)$
R: $[0, \infty)$

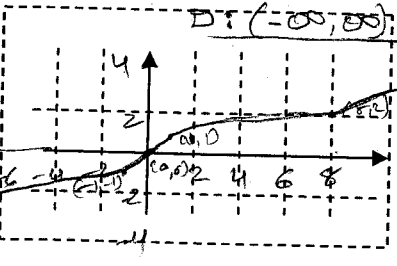


moves 2 PE down when constant outside of Root

x	y
4	0
5	1
9	2
16	3
25	4

c. $f(x) = \sqrt[3]{x}$ include negative part. D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

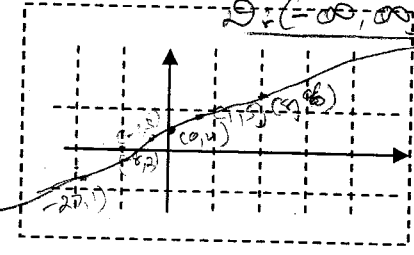
x	y
0	0
1	1
8	2
27	3
-1	-1
-8	-2
-27	-3



x	y
0	0
1	1
8	2
27	3
-1	-1
-8	-2
-27	-3

d. $f(x) = \sqrt[3]{x+4}$ D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

goes up 4 point on y-axis



2. Use the square root property to solve each equation and give the solution set.

a) $x^2 = 49$

$x = \pm\sqrt{49}$
 $x = \pm 7$

sol. set $\{x | x = \pm 7\}$

c) $(x-4)^2 = 25$

$x-4 = \pm\sqrt{25}$

$x-4 = \pm 5$

$x-4 = 5$ $x-4 = -5$

$x = 9$ $x = -1$

sol. set $\{x | x = 9 \text{ or } x = -1\}$

b) $x^2 - 8 = 0$

$x^2 = 8$

$x = \pm\sqrt{8}$ (une)

$x = \pm 2\sqrt{2}$

sol. set = $\{x | x = \pm 2\sqrt{2}\}$

d) $(2x-5)^2 = 12$

$2x-5 = \pm\sqrt{12}$

$2x-5 = \pm 2\sqrt{3}$

$2x-5 = 2\sqrt{3}$

$2x = 5 + 2\sqrt{3}$

$x = \frac{5 + 2\sqrt{3}}{2}$

$2x-5 = -2\sqrt{3}$

$2x = 5 - 2\sqrt{3}$

$x = \frac{5 - 2\sqrt{3}}{2}$

sol. set $\{x | x = \frac{5 \pm 2\sqrt{3}}{2}\}$

3. Use the cube roots to solve each equation and give the solution set.

a) $x^3 = 125$

$x = \sqrt[3]{125}$

$x = 5$ or $x = 5$

b) $\frac{3(y-1)^3}{3} = \frac{81}{3}$

$y-1 = \sqrt[3]{27}$

$y-1 = 3$

$y = 4$

sol. set $\{x | x = 4\}$

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GPS # 36

7.6 COMPLEX NUMBERS

NAME: Parul Patel

Useful Guidelines:

- * Imaginary Unit: $i = \sqrt{-1}$ and $i^2 = -1$. Example: $\sqrt{-5} = i\sqrt{5}$. i = imaginary number
- * Standard Form: $a + bi$, where a and b are real numbers.
- * Real Part of $a + bi$: a
- * Imaginary Part of $a + bi$: b
- * Complex conjugate of $a + bi$: $a - bi$

1. Use the imaginary unit to write the expression.

a. $\sqrt{-36} = \sqrt{(-1)(36)}$

c. $\sqrt{-72} = \sqrt{(-1) \sqrt{36}} = i6$

$\sqrt{(-1)(36)2} = 6\sqrt{2}i$

2. Write each sum or difference in standard form.

a. $(2+i) + (9-2i)$

$= 11 - i$

c. $(-3+2i) + (4-4i)$

$= -3+2i+4-4i = 1-2i$

3. Write each product in standard form.

a. $(3-i)(2+2i)$

$= 6 - 2i + 6i - 2i^2$

$= 6 + 4i - 2(-1)$

$= 8 + 4i$ ← imaginary number

c. $(-4+3i)(2+8i)$

$= -12 + 6i - 32i + 24i^2$

$= -12 - 26i + 24(-1)$

$= -36 - 26i$

4. Write each quotient in standard form.

a. $\frac{3(4+3i)}{4-3i(4+3i)}$

$= \frac{12+9i}{16-9i^2}$

$= \frac{12+9i}{16-9(-1)}$

$= \frac{12+9i}{25}$
 $= \frac{12}{25} + \frac{9i}{25}$
 ↓ ↑
 Real # Imaginary #

c. $\frac{2-i}{3+4i} \cdot \frac{3-4i}{3-4i}$

$= \frac{6-3i-8i+4i^2}{9-16i^2}$

$= \frac{6-11i+4(-1)}{9+16}$

$= \frac{2-11i}{25}$

$= \frac{2}{25} - \frac{11}{25}i$

b. $\frac{2i(2-5i)}{2+5i(2-5i)}$

$= \frac{4i-10i^2}{4-25i^2}$

$= \frac{4i-10(-1)}{4+25}$

$= \frac{4i+10}{29}$

d. $\frac{(1+2i)^2 - 5i}{5i(1-5i)}$

$= \frac{-5i-10i^2}{-25i^2}$

$= \frac{-5i-10(-1)}{-25(-1)}$

$= \frac{10}{29} + \frac{4}{29}i$

$= \frac{-5i+10}{25}$

$= \frac{10}{25} - \frac{5i}{25}$

$= \frac{2}{5} - \frac{1}{5}i$