INTERMEDIATE ALGEBRA

GPS # 37 **8.1 QUADRATIC FUNCTIONS AND THEIR GRAPHS** **NAME:** Antoinette Durden

Useful Guidelines:

* Quadratic Function: $f(x) = ax^2 + bx + c, (a \neq 0)$, where a, b and c are real numbers.

* The vertex of the graph of $f(x) = ax^2 + bx + c, (a \neq 0)$ has coordinates $(x, y) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

* To graph a quadratic function:
  - Step 1: Determine whether the parabola opens up or down.
    - [If $a > 0$, the parabola is open up; If $a < 0$, the parabola is open down.]
  - Step 2: Find the vertex.
  - Step 3: Find the x-intercepts (if any) and y-intercept.
  - Step 4: Plot the graph. [Find additional points as needed.]

Good job.

Graph each parabola, Give the vertex, axis, domain, and range.

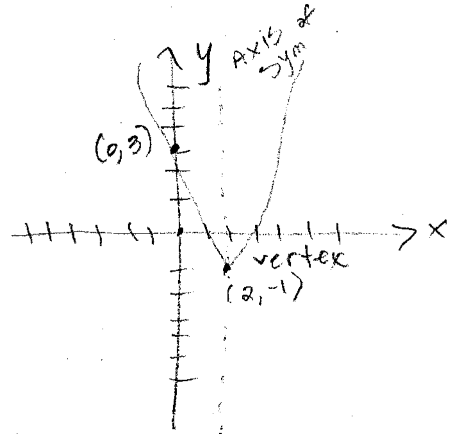
1. $f(x) = x^2 - 4x + 3$ $a=1$ $b=-4$ $c=3$ $a>0$ ∪

$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

$x = 2$

$f(2) = (2)^2 - 4(2) + 3 = -1$

vertex $= (2, -1)$
x, y



y intercept
Let $x = 0$
$f(0) = 3$ $(0,3)$
x intercept
let $y = 0$
$(x^2 - 4x + 3) = 0$
$(x-1)(x-3) = 0$
$x = 1, 3$

D: $(-\infty, \infty)$
R: $[-1, \infty)$

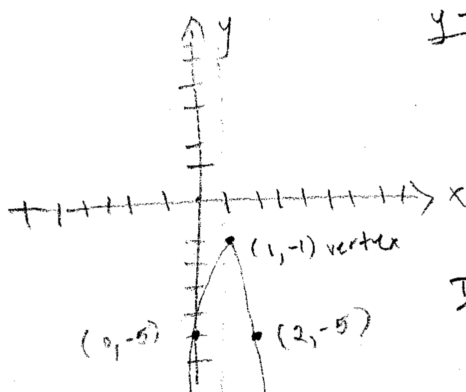
2. $f(x) = -4x^2 + 8x - 5$ ∩ $a = -4 < 0$ $b = 8$ $c = -5$

$x = \frac{-b}{2a} = \frac{-(8)}{2(-4)} = 1$

$x = 1$

$f(1) = -4(1)^2 + 8(1) - 5 = -1$

vertex $= (1, -1)$
x, y



y-intercept
Let $x = 0$
$f(0) = -5$ $(0, -5)$
x-intercept
Let $y = 0$
$-4x^2 + 8x - 5 = 0$
$(2x+1)(2x+5) = 0$
$2x + 1 = 0$
$2x = -1$
$x = -\frac{1}{2}$
$-2x = -5$
$x = \frac{5}{2}$
$x = -\frac{1}{2}, \frac{5}{2}$

D: $(-\infty, \infty)$
R: $(-\infty, -1]$

3. A rocket is fired upward. After x hour, the height of the rocket is given by $f(x) = -16x^2 + 32x$. Find the time required in hours for the rocket to reach maximum height, and find the maximum height in kilometers. $a = -16 < 0$ ∩ $b = 32$ $c = \emptyset$

$x = \frac{-b}{2a} = \frac{-32}{2(-16)} = 1$

$x = 1$ hour

$f(1) = -16(1)^2 + 32(1) = 16$

$f(1) = 16$ km max height

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GPS # 38 **8.2 PARABOLAS AND MODELING** **NAME:** Antoinette Durden

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Useful Guidelines:

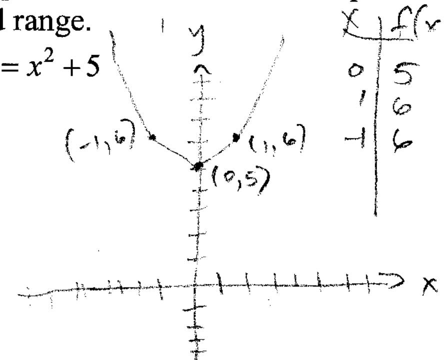
* $f(x) = x^2 + k$: A parabola with the same shape as the graph of $f(x) = x^2$. The parabola is shifted vertically k units up if $k > 0$ or k units down if $k < 0$; Vertex: $(0,k)$.
* $f(x) = (x-h)^2$: A parabola with the same shape as the graph of $f(x) = x^2$. The parabola is shifted horizontally h units to the right if $h > 0$ or h units to the left if $h < 0$; Vertex: $(h,0)$.

Graph each parabola. Plot at least two points in addition to the vertex. Give the vertex, axis, domain, and range.

1. a) $f(x) = x^2 + 5$

vertex: (0,5)
D: (-∞, ∞)
R: [5, ∞)
axis x=0

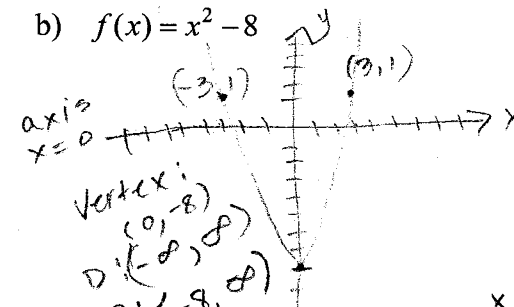
x	f(x) = x²+5
0	5
1	6
-1	6



b) $f(x) = x^2 - 8$

x	f(x) = x²-8
0	-8
-3	1
3	1

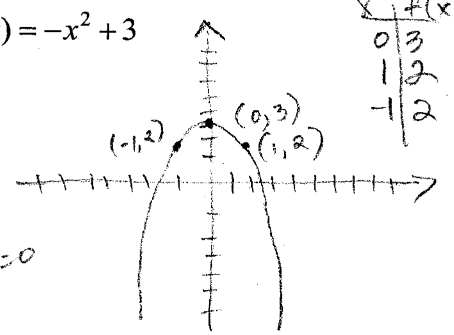
axis x=0
vertex: (0,-8)
D: (-∞, ∞)
R: [-8, ∞)



2. a) $f(x) = -x^2 + 3$

vertex: (0,3)
D: (-∞, ∞)
R: (-∞, 3]
axis x=0

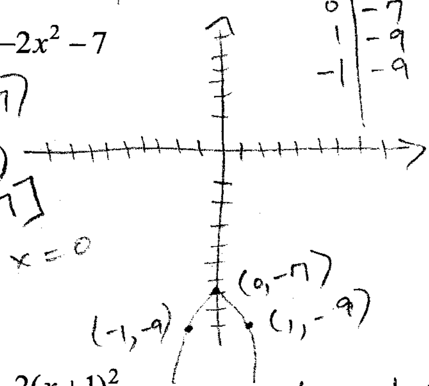
x	f(x) = -x+3
0	3
1	2
-1	2



b) $f(x) = -2x^2 - 7$

vertex: (0,-7)
D: (-∞, ∞)
R: (-∞, -7]
axis x=0

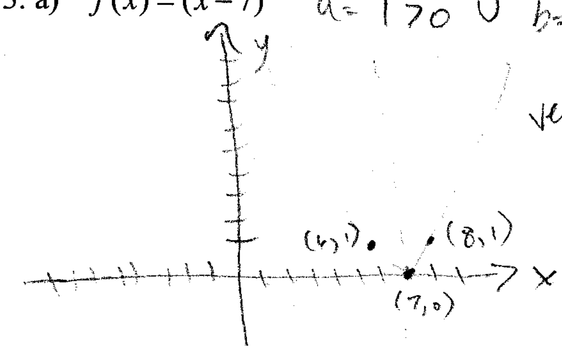
x	f(x)
0	-7
1	-9
-1	-9



3. a) $f(x) = (x-7)^2$ a = 1 b = -14 c = 49

x	f(x)
7	0
6	1
8	1

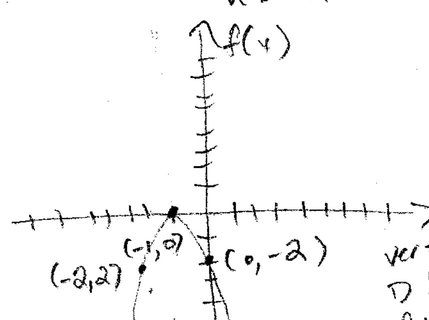
vertex (7,0)
D: (-∞, ∞)
R: [0, ∞)



b) $f(x) = -2(x+1)^2$ a = -2 b = -4 c = -2

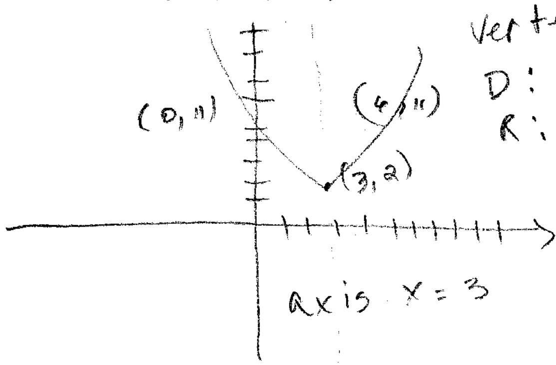
x	f(x)
-1	0
0	-2
-2	-2

vertex (-1,0)
D: (-∞, ∞)
R: (-∞, 0]



4. a) $f(x) = (x-3)^2 + 2$

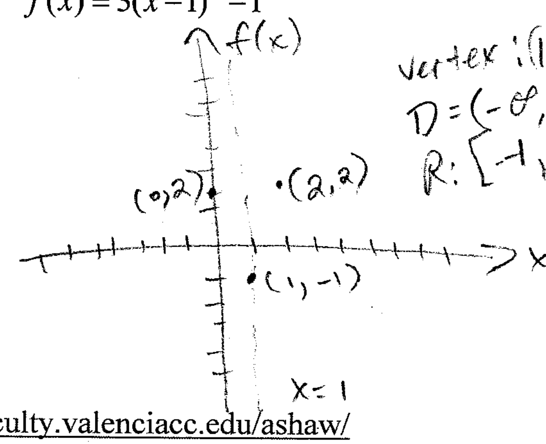
vertex: (3,2)
D: (-∞, ∞)
R: [2, ∞)



axis x=3

b) $f(x) = 3(x-1)^2 - 1$

vertex: (1,-1)
D = (-∞, ∞)
R: [-1, ∞)



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GPS # 39 8.3 QUADRATIC EQUATIONS NAME: Antoinette Durden

Useful Guidelines:

* Quadratic Equation: $f(x) = ax^2 + bx + c = 0 (a \neq 0)$, where *a*, *b* and *c* are real numbers.

* Square Root Property: If *x* and *k* are complex numbers and $x^2 = k$, then $x = \sqrt{k}$ or $x = -\sqrt{k}$.

* Completing the Square: To solve $ax^2 + bx + c = 0 (a \neq 0)$:

Step 1: If $a \neq 1$, divide each side by *a*.

Step 2: Write the equation with the variable terms on one side and the constant on the other.

Step 3: Take half the coefficient of *x* and square it.

Step 4: Add the square to both sides of the equation.

Step 5: Factor the perfect square of a trinomial, write it as the square of a binomial, and simplify.

Step 6: Use the square root property to complete the solution and write down the solution set.

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Use the square root property to solve each equation and give the solution set.

1. a) $x^2 = 49$

$X = \pm\sqrt{49}$

$X = \pm 7$

$\{x \mid x = \pm 7\}$

b) $x^2 - 8 = 0$

$X^2 = 8$

$X = \pm\sqrt{8}$

$X = \pm\sqrt{(4)(2)}$

$X = \pm 2\sqrt{2}$

$\{x \mid x = \pm 2\sqrt{2}\}$

c) $(x-4)^2 = 25$

$X - 4 = \pm\sqrt{25}$

$X - 4 = \pm 5$

$X = 4+5 \mid X = 4-5$

$X = 9 \mid X = -1$

$\{x \mid x = -1 \text{ or } x = 9\}$

d) $(2x-5)^2 = 12$

$2X - 5 = \pm\sqrt{12}$

$2x - 5 = \pm\sqrt{(4)(3)}$

$2x - 5 = \pm 2\sqrt{3}$

$2x = 2\sqrt{3} + 5$ or $2x = -2\sqrt{3} + 5$

$\{x = \frac{5}{2} \pm \sqrt{3}\}$

Solve each equation by completing the square and give the solution set.

2. a) $\frac{2x^2 + 8x + 2}{2} = \frac{0}{2}$

$a = 2 \quad b = 8 \quad c = 2$

① $a = 1$ $\quad X^2 + 4x + 1 = 0$

② $\quad X^2 + 4x + \boxed{4} = -1 + \boxed{4}$

③ $\left(\frac{4}{2}\right)^2 = 4$

$(x+2)^2 = 3$

$x + 2 = \pm\sqrt{3}$

$X = -2 + \sqrt{3}$ or $X = -2 - \sqrt{3}$

Sol set $\{-2 \pm \sqrt{3}\}$

b) $z^2 - 10z + 15 = 0$

$a = 1 \quad b = 10 \quad c = 15$

② $\quad z^2 - 10z + \boxed{25} = -15 + \boxed{25}$

③ $\left(\frac{10}{2}\right)^2$

$(x-5)^2 = 10$

$x - 5 = \pm\sqrt{10}$

$X = 5 + \sqrt{10}$ or $X = 5 - \sqrt{10}$

Sol set $\{X = 5 \pm \sqrt{10}\}$

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GPS # 40 8.4 THE QUADRATIC FORMULA NAME: Antoinette Durden

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Useful Guidelines:

The solutions of $ax^2+bx+c=0 (a \neq 0)$ are given by $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$. "Quadratic Formula"

If a, b, and c are integers, then the number and type of solutions can be predicted as follows:

* If the discriminant $b^2-4ac>0$, then we'll have two real solutions.
* If the discriminant $b^2-4ac=0$, then we'll have only one real solution.
* If the discriminant $b^2-4ac<0$, then we'll have two complex solutions.

1. Solve each equation using the quadratic formula and give the solution set.

a) $x^2-x-12=0$ $a=1$ $b=-1$ $c=-12$

$X=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $X=\dfrac{-(-1)\pm\sqrt{(-1)^2-(4)(1)(-12)}}{(2)(1)}$

$X=\dfrac{1\pm\sqrt{1+48}}{2}$ $X=\dfrac{1\pm\sqrt{49}}{2}$ $X=\frac{1}{2}+\frac{7}{2}=\frac{8}{2}=4$

$X=\frac{1}{2}-\frac{7}{2}-\frac{6}{2}=-3$

$\{-3,4\}$

b) $y^2-6y-8=0$ $a=1$ $b=-6$ $c=-8$

$X=\dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(-8)}}{2(1)}$

$X=\dfrac{6\pm\sqrt{36+32}}{2}$

$X=\dfrac{6\pm\sqrt{68}}{2}=\dfrac{6\pm 2\sqrt{12}}{2}=$

$3\pm\sqrt{17}$

c) $2x^2-3x+3=0$ $a=2$ $b=-3$ $c=3$

$X=\dfrac{-(-3)\pm\sqrt{(-3)^2-4(2)(3)}}{2(2)}$

$X=\dfrac{3\pm\sqrt{9-24}}{4}=$ $X=\dfrac{3\pm\sqrt{-15}}{4}$

$X=\dfrac{3\pm i\sqrt{15}}{4}$

a+bi $\dfrac{3}{4}\pm\dfrac{\sqrt{15}}{4}i$

d) $4x^2-5x+2=0$ $a=4$ $b=-5$ $c=2$

$X=\dfrac{-(-5)\pm\sqrt{(-5)^2-4(4)(2)}}{2(4)}$

$X=\dfrac{5\pm\sqrt{25-32}}{8}$ $\dfrac{5}{8}\pm\dfrac{\sqrt{7}}{8}i$

$X=\dfrac{5\pm\sqrt{-7}}{8}=\dfrac{5\pm i\sqrt{7}}{8}$

2. Use the discriminant to predict whether the solutions to each equation are
A. *one real solution*; B. *two real solutions*; C. *two complex solutions*.

a) $x^2+5x+4=0$ $a=1$ $b=5$ $c=4$

b^2-4ac
$=25-4(1)(4)$
$=25-16$
$=9$

$9>0$ 2 real solutions

(B)

b) $2x^2-4x+2=0$ $a=2$ $b=-4$ $c=2$

b^2-4ac
$=16-4(2)(2)$
$=16-16$
$=0$

(A) $0=0$ 1 real Solution

c) $5x^2-3x+7=0$ $A=5$ $b=-3$ $c=7$

$=9-4(5)(7)$
$=9-140$
$=-131<0$

(C) 2 complex Solutions

d) $x^2+3x-1=0$