

# INTERMEDIATE ALGEBRA

GPS #37

8.1 QUADRATIC FUNCTIONS AND THEIR GRAPHS NAME: Poojai Patel

**Useful Guidelines:** as long as highest power 2

\* Quadratic Function:  $f(x) = ax^2 + bx + c, (a \neq 0)$ , where  $a, b$  and  $c$  are real numbers.

\* The vertex of the graph of  $f(x) = ax^2 + bx + c, (a \neq 0)$  has coordinates  $(x, y) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ .

To graph a quadratic function:

Step 2: Determine whether the parabola opens up or down.

[If  $a > 0$ , the parabola is open up; If  $a < 0$ , the parabola is open down.]

Step 3: Find the vertex. (Location of  $(x, y)$ )

Step 4: Find the x-intercepts (if any) and y-intercept.

Step 5: Plot the graph. [Find additional points as needed.]

Graph each parabola, Give the vertex, axis, domain, and range.

1.  $f(x) = x^2 - 4x + 3$

Step 1:  $a = 1 > 0$ , U facing up.

Step 3: Find vertex  $x = -\frac{b}{2a} = \frac{-(-4)}{2(1)} = 2$

$b = -4$   
 $c = 3$

$f(2) = (2)^2 - 4(2) + 3$   
 $= 4 - 8 + 3$   
 $= -1$

So,  $(x, y)$  is  $(2, -1)$

①  $a > 0$  U  
 $a < 0$  ∩

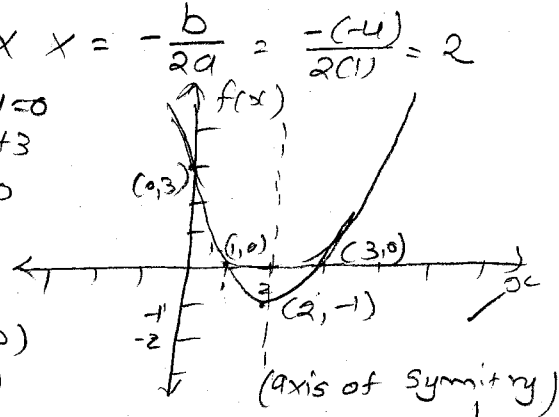
③ y-int - Let  $x = 0$

$f(0) = 3$   
 $(0, 3)$

② Vertex  $(2, -1)$

x-int - Let  $y = 0$   
 $0 = x^2 - 4x + 3$   
 $(x-1)(x-3) = 0$   
 $x = 1, x = 3$

$D = (-\infty, \infty)$   
 $R = [-1, \infty)$



2.  $f(x) = -4x^2 + 8x - 5$

Step 1:  $a = -4, b = 8, c = -5$

Step 2:  $a < 0$  parabola ∩ facing down.

Step 3: vertex  $x = -\frac{b}{2a} = \frac{-8}{2(-4)} = \frac{-8}{-8} = 1$

Step 5: Find x & y intercept

→ y-intercept  
Let  $x = 0$

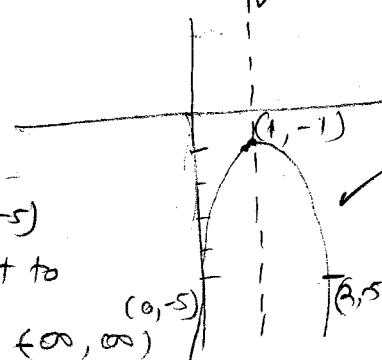
$y = -4(0)^2 + 8(0) - 5$   
 $= 0 + 0 - 5$

$y = -5$  y-intercept  $(0, -5)$

→ Find second point opposite to

$(0, -5) \rightarrow (2, -5)$

$D = (-\infty, \infty)$   
 $R = (-\infty, -1]$



3. A rocket is fired upward. After  $x$  hour, the height of the rocket is given by  $R = -16x^2 + 32x$ . Find the time required in hours for the rocket to reach maximum height, and find the maximum height in kilometers.

$a = -16$  so ∩

$f(x) = -16(1)^2 + 32(1)$

$b = 32$

$y = 16$  k.m. high

$c = 0$

Vertex  $x = \frac{-b}{2a}$

So, vertex  $(x, y)$

$= \frac{-32}{2(-16)} = \frac{-32}{-32}$

$= (1, 16)$

$t = 1$  hours

R0907 a.s.

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8.2 PARABOLAS AND MODELING

NAME:

Pamul Patel

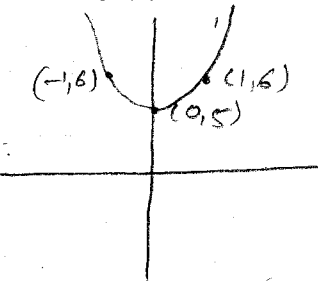
**Useful Guidelines:**

\*  $f(x) = x^2 + k$ : A parabola with the same shape as the graph of  $f(x) = x^2$ . The parabola is shifted vertically  $k$  units up if  $k > 0$  or  $k$  units down if  $k < 0$ ; Vertex:  $(0, k)$ .

\*  $f(x) = (x - h)^2$ : A parabola with the same shape as the graph of  $f(x) = x^2$ . The parabola is shifted horizontally  $h$  units to the right if  $h > 0$  or  $h$  units to the left if  $h < 0$ ; Vertex:  $(h, 0)$ .

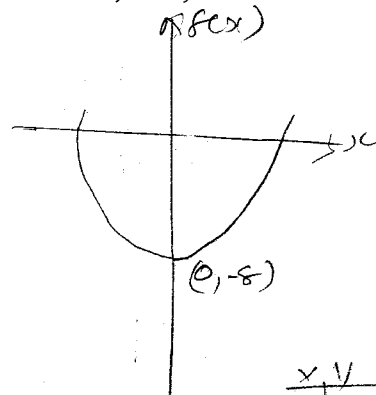
Graph each parabola. Plot at least two points in addition to the vertex. Give the vertex, axis, domain, and range.

1. a)  $f(x) = x^2 + 5$



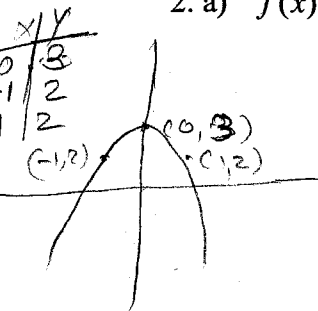
Vertex =  $(0, 5)$   
 $D = (-\infty, \infty)$   
 $R = [5, \infty)$   
 Axis of Symmetry  $x = 0$

b)  $f(x) = x^2 - 8$



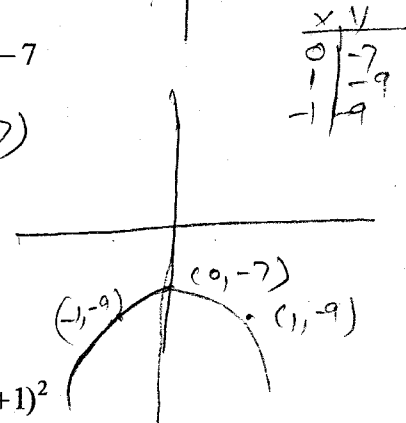
Vertex =  $(0, -8)$   
 $D = (-\infty, \infty)$   
 $R = [-8, \infty)$

2. a)  $f(x) = -x^2 + 3$



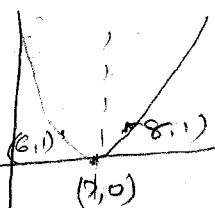
Vertex  $(0, 3)$   
 $D = (-\infty, \infty)$   
 $R = (-\infty, 3]$

b)  $f(x) = -2x^2 - 7$



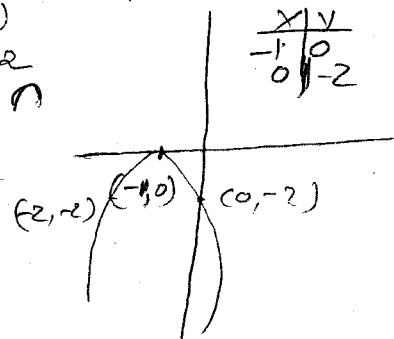
Vertex =  $(0, -7)$   
 $D = (-\infty, \infty)$   
 $R = (-\infty, -7]$

3. a)  $f(x) = (x-7)^2$  axis  $> 0$  so  $\cup$   
 $(x-7)(x-7) = x^2 - 14x + 49$



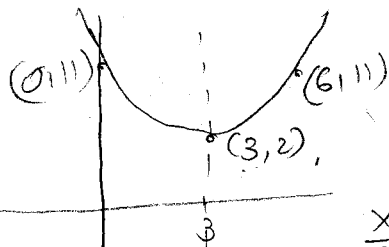
Vertex:  $(7, 0)$   
 $D = (-\infty, \infty)$   
 $R = [0, \infty)$   
 Axis:  $x = 7$

b)  $f(x) = -2(x+1)^2$   
 $= -2(x^2 + 2x + 1)$   
 $= -2x^2 - 4x - 2$   
 $a = -2$  going  $\cap$   
 $b = -4$   
 $c = -2$



Vertex  $(-1, 0)$   
 $D = (-\infty, \infty)$   
 $R = (-\infty, 0]$   
 axis  $x = -1$

4. a)  $f(x) = (x-3)^2 + 2$

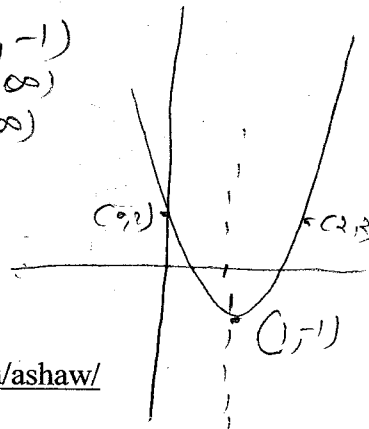


Vertex  $(3, 2)$   
 $D = (-\infty, \infty)$   
 $R = [2, \infty)$   
 axis  $x = 3$

x	y
0	11

x	y
0	2

Vertex:  $(1, -1)$   
 $D = (-\infty, \infty)$   
 $R = [-1, \infty)$   
 axis  $x = 1$



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8.3 QUADRATIC EQUATIONS

NAME:

Pooj Patel

## Useful Guidelines:

\* Quadratic Equation:  $f(x) = ax^2 + bx + c = 0 (a \neq 0)$ , where  $a, b$  and  $c$  are real numbers.

\* Square Root Property: If  $x$  and  $k$  are complex numbers and  $x^2 = k$ , then  $x = \sqrt{k}$  or  $x = -\sqrt{k}$ .

\* Completing the Square: To solve  $ax^2 + bx + c = 0 (a \neq 0)$ :

Step 1: If  $a \neq 1$ , divide each side by  $a$ .

Step 2: Write the equation with the variable terms on one side and the constant on the other.

Step 3: Take half the coefficient of  $x$  and square it.  $(\frac{b}{2})^2$

Step 4: Add the square to both sides of the equation.

Step 5: Factor the perfect square of a trinomial, write it as the square of a binomial, and simplify.

Step 6: Use the square root property to complete the solution and write down the solution set.

Use the square root property to solve each equation and give the solution set.

1. a)  $x^2 = 49$

$x = \pm \sqrt{49}$

$x = \pm 7$

Sol. set:  $\{x | x = \pm 7\}$

c)  $(x-4)^2 = 25$

$x-4 = \pm \sqrt{25}$

$x-4 = \pm 5$

$x-4=5$  or  $x-4=-5$

$x=9$

$x=-1$

Sol. set:  $\{x | x = -1 \text{ or } x = 9\}$

Solve each equation by completing the square and give the solution set.

2. a)  $2x^2 + 8x + 2 = 0$

$a=2, b=8, c=2$   
 $2(x^2 + 4x + 1) = 0$

step ①  $x^2 + 4x + 1 = 0/2$

step ②  $x^2 + 4x + 4 = -1 + 4$

step ③  $(x+2)^2 = 3$

step ④  $x+2 = \pm \sqrt{3}$

$x+2 = -\sqrt{3}$  or  $x+2 = \sqrt{3}$

$x = -2 - \sqrt{3}$  or  $x = -2 + \sqrt{3}$

Sol. set  $\{-2 \pm \sqrt{3}\}$

b)  $z^2 - 10z + 15 = 0$

$a=1, b=-10, c=15$   
 step ①  $z^2 - 10z + 25 = -15 + 25$

step ②  $(z-5)^2 = 10$

step ③  $(z-5)^2 = 10$

step ④  $z-5 = \pm \sqrt{10}$

Sol. set  $\{5 \pm \sqrt{10}\}$

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8.4 THE QUADRATIC FORMULA

NAME: Parul Patel

## Useful Guidelines:

The solutions of  $ax^2 + bx + c = 0 (a \neq 0)$  are given by  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . "Quadratic Formula"

If  $a$ ,  $b$ , and  $c$  are integers, then the number and type of solutions can be predicted as follows:

- \* If the discriminant  $b^2 - 4ac > 0$ , then we'll have two real solutions.
- \* If the discriminant  $b^2 - 4ac = 0$ , then we'll have only one real solution.
- \* If the discriminant  $b^2 - 4ac < 0$ , then we'll have two complex solutions.

1. Solve each equation using the quadratic formula and give the solution set.

$a=1$   
 $b=-1$   
 $c=-12$

a)  $x^2 - x - 12 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-12)}}{2(1)} = \frac{1 \pm 7}{2} = \frac{8}{2} \text{ or } \frac{-6}{2}$$

$$x = \frac{1 \pm \sqrt{1+48}}{2} = 4 \text{ or } -3$$

Sol. set  $\{4, -3\}$

b)  $y^2 - 6y - 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{36 + 32}}{2} = 3 \pm \sqrt{17}$$

$$= \frac{6 \pm \sqrt{68}}{2} = \frac{6 \pm 2\sqrt{17}}{2}$$

Sol. set  $\{3 \pm \sqrt{17}\}$

$a=1$   $b=-6$   $c=-8$

c)  $2x^2 - 3x + 3 = 0$

$a=2$   $b=-3$   $c=3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{3 \pm \sqrt{9 - 24}}{4}$$

$$= \frac{3 \pm \sqrt{-15}}{4}$$

$$= \frac{3 \pm \sqrt{-1} \cdot \sqrt{15}}{4}$$

$$= \frac{3 \pm i\sqrt{15}}{4} = \frac{3}{4} \pm \frac{\sqrt{15}}{4}i$$

Sol. set  $\left\{ \frac{3}{4} \pm \frac{\sqrt{15}}{4}i \right\}$

d)  $4x^2 - 5x + 2 = 0$   $a=4$   $b=-5$   $c=2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25 - 32}}{8} = \frac{5}{8} \pm \frac{\sqrt{-7}}{8}i$$

$$= \frac{5 \pm \sqrt{-7}}{8}$$

$$= \frac{5 \pm \sqrt{7}i}{8}$$

$$= \frac{5}{8} \pm \frac{\sqrt{7}}{8}i$$

Sol. set  $\left\{ \frac{5}{8} \pm \frac{\sqrt{7}}{8}i \right\}$

2. Use the discriminant to predict whether the solutions to each equation are

A. one real solution; B. two real solutions; C. two complex solutions.

a)  $x^2 + 5x + 4 = 0$

$a=1$   $b=5$   $c=4$

$$b^2 - 4ac$$

$$= 25 - 4(1)(4)$$

$$= 25 - 16 = 9 > 0 \text{ 2 real sol.}$$

c)  $5x^2 - 3x + 7 = 0$

$a=5$   $b=-3$   $c=7$

$$= b^2 - 4ac$$

$$= 9 - 140$$

$$= -131 < 0 \text{ so, 2 complex}$$

R0907 a.s.

Sol.

Ans: **C**

b)  $2x^2 - 4x + 2 = 0$

$a=2$   $b=-4$   $c=2$

$$b^2 - 4ac$$

$$= 16 - 16$$

$$= 0 = 0 \text{ so 1 real sol. Ans } \textcircled{A}$$

d)  $x^2 + 3x - 1 = 0$

$a=1$   $b=3$   $c=-1$

$$b^2 - 4ac$$

$$= 9 + 4$$

$$= 13 > 0 \text{ so, 2 real sol.}$$

Ans: **B**