INTERMEDIATE ALGEBRA

Useful Definitions and Rules: For all integers m and n and all real numbers a and b:

* Exponential Expression: $2^4 = 2 \cdot 2 \cdot 2 \cdot 2$ (4 factors of 2), $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ (5 factors of 3),

  $a^5 = a \cdot a \cdot a \cdot a \cdot a$ (5 factors of a), $a^n = a \cdot a \cdot a \cdot a \cdot a...a$ (n factors of a)

* Product Rule: $a^m \cdot a^n = a^{m+n}$ * Zero Exponent: $a^0 = 1 (a \neq 0)$

* Quotient Rule: $\dfrac{a^m}{a^n} = a^{m-n}$ * Negative Exponent: $a^{-n} = \dfrac{1}{a^n} (a \neq 0)$

* Power Rules: $(a^m)^n = a^{mn}$; $(ab)^m = a^m b^m$

  $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n} (b \neq 0)$

A number is in <u>scientific notation</u> when it is written as a product of a number between 1 and 10 (inclusive of 1) and an integer power of 10. For example: 2.35×10^{10}

Assume all variables represent nonzero real numbers for the following:

1. Evaluate the following:

a) $2^3 = 2 \cdot 2 \cdot 2 = 8$

b) $(-3)^4 = -3 \cdot -3 \cdot -3 \cdot -3 = +9 \cdot +9 = 81$

c) $-4^3 = -(4)(4)(4) = -64$

d) $\sqrt{25} = \sqrt{5^2} = (5^2)^{\frac{1}{2}} = 5$

e) $\sqrt[3]{8} = (2^3)^{\frac{1}{3}} = 2$

f) $\sqrt{\dfrac{16}{9}} = \dfrac{\sqrt{16}}{\sqrt{9}} = \dfrac{4^{2 \cdot \frac{1}{2}}}{3^{2 \cdot \frac{1}{2}}} = \dfrac{4}{3}$

2. Apply the product rule for exponents, if possible, in each case.

a) $4^5 \cdot 4^2 = 4^{(5+2)} = 4^7$ Product

b) $7^3 \cdot 7^9 \cdot 7^2 = 7^{(3+9+2)} = 7^{14}$

c) $(-4x^3)(6x^2) = -4 \cdot x^3 \cdot 6 \cdot x^2 = -24x^5$

d) $(2x^5)(y^2) = 2x^5 y^2$

3. Apply the quotient rule for exponents and write each result using only positive exponents.

a) $\dfrac{2^5}{2^3} = 2^{(5-3)} = 2^2 = 4$ Quotient Rule

b) $\dfrac{7^{-2}}{7^8} = 7^{(-2-8)} = 7^{-10} = \dfrac{1}{7^{10}}$

c) $\dfrac{x^8}{x^3} = x^{(8-3)} = x^5$

d) $\dfrac{y^4}{y^{-2}} = y^{(4-(-2))} = y^6$

3. Use one or more power rules to simplify each expression.

a) $(y^5)^2 = y^{10}$ Power Rule

b) $(x^{-2})^3 = x^{-6} = \dfrac{1}{x^6}$

c) $(3x)^4 = 3^4 \cdot x^4 = 81x^4$

d) $\left(\dfrac{6}{-5}\right)^2 = \dfrac{6^2}{-5^2} = \dfrac{36}{25}$

4. Evaluate and simplify each expression so that no negative exponents appear in the final result

a) $-(-7)^0 = -(1) = -1$ Zero Exponent

b) $2x^0 - y^0 = 2(1) - 1 = 2 - 1 = 1$

c) $(2k^{-1})^4 = \left(\dfrac{2}{k^1}\right)^4 = \dfrac{2^4}{k^4} = \dfrac{16}{k^4}$ Negative Exponent

d) $\left(\dfrac{6x^{-2}}{x^{-3}}\right)^2 = \left(\dfrac{6x^3}{x^2}\right)^2 = (6x^{3-2})^2 = (6x^1)^2 = 36x^2$

5. Write each number in scientific notation.

a) $3600 = 3.6 \times 10^3$

b) $-790,000 = -7.9 \times 10^5$

c) $0.000896 = 8.96 \times 10^{-4}$

d) $-0.0000555 = -5.55 \times 10^5$