INTERMEDIATE ALGEBRA

GPS # 30 **7.1 RADICAL EXPRESSIONS AND RATIONAL EXPONENTS** **NAME:** Kelly Fenton

Good Job!

Useful Guidelines:

* $\sqrt[n]{a} = a^{\frac{1}{n}} = b$ means $a = b^n$, where $\sqrt[n]{a}$ is the principal n^{th} root of a.
* $\sqrt[n]{a^n} = |a|$ if n is even. For example: $\sqrt[4]{(-3)^4} = |(-3)| = 3$.
* $\sqrt[n]{a^n} = a$ if n is odd. For example: $\sqrt[3]{(-5)^3} = -5$.

[Remember: $(a^n)^m = a^{nm}$, $a^m a^n = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$]

Rational Exponents:

* If m and n are positive integers with $\frac{m}{n}$ in lowest term, then $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$.

[Remember: $(ab)^m = a^m b^m$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$]

Simplify each root so that no radicals appear. Assume all variables represents real numbers.

1. a) $\sqrt{81} = 9$ b) $\sqrt[3]{8} = (2^3)^{\frac{1}{3}} = 2$ c) $\sqrt[5]{32} = (2^5)^{\frac{1}{5}} = 2$

2 27 3 81

even root = absolute value

d) $\sqrt[4]{(-3)^4} = 3$

odd root = negative value

e) $\sqrt[3]{(-5)^3} = -(-5) = 5$

Positive root = positive value

f) $-\sqrt[4]{(-2)^4} = -(2) = -2$

always assume ≥ 0

2. a) $\sqrt{5^2} = \sqrt{5^2} = 5$ b) assume ≥ 0 $\sqrt{(-5)^2} = \sqrt{(-5)^2} = 5$ even root = positive value $= |-5|$ c) $\sqrt[6]{(-5)^6} = 5$ $= |-5|$ d) $\sqrt{x^2} = |x|$ $= |x|$ (always keep ab. value)

e) $\sqrt[3]{x^9} = (x^9)^{\frac{1}{3}} = x^3$ f) $-\sqrt[3]{27t^3}$ $\sqrt[3]{(3t)^3} = -3t$ g) $\sqrt[5]{32k^5}$ $= \sqrt[5]{(2k)^5} = 2k$ h) $-\sqrt{x^2} = -|x|$

Evaluate the following, simplify if possible, and write the answer with only positive exponents.

3. a) $8^{\frac{4}{3}}$ $= (2^3)^{\frac{4}{3}}$ $= (2)^4$ $= 16$

b) $(16)^{\frac{3}{4}}$ $= (2^4)^{\frac{3}{4}}$ $= (2)^3$ $= 8$

c) $\left(\frac{1000}{27}\right)^{-4/3}$ = flip b/c neg. exp. $= \left(\frac{27}{1000}\right)^{\frac{4}{3}}$ $= \frac{(3^3)^{4/3}}{(10^3)^{4/3}} = \frac{(3)^4}{(10)^4} = \frac{81}{10{,}000}$

d) $-\left(\frac{16}{81}\right)^{\frac{3}{4}} = -\frac{(2^4)^{\frac{3}{4}}}{(3^4)^{\frac{3}{4}}}$ $= -\frac{(2)^3}{(3)^3}$ $= -\frac{8}{27}$

4. a) $3^{\frac{1}{3}} \cdot 3^{\frac{5}{3}}$ $3^{\frac{1}{3}} \cdot 3^{\frac{5}{3}} = 3^{\frac{1}{3}+\frac{5}{3}} = 3^{\frac{6}{3}} = 3^2$

b) $x^{-\frac{1}{2}}(x^{\frac{5}{2}} + x)$ distribute $= x^{-\frac{1}{2}} \cdot x^{\frac{5}{2}} + x^{-\frac{1}{2}} \cdot x^1$ $= x^2 + x^{\frac{1}{2}}$ $= x^2 + \sqrt{x}$

5 a) $\frac{t^{5/6} \cdot t^{-2/6}}{t} = \frac{t^{\frac{3}{6}}}{t^1} = t^{\frac{1}{2}-1} = t^{-\frac{1}{2}} = \frac{1}{t^{\frac{1}{2}}}$ $= \frac{1}{\sqrt{t}}$

xb) $\sqrt[3]{x^4} \cdot \sqrt[3]{x^6} = \sqrt[3]{x^{10}}$