

**Useful Guidelines:**

- \* **One-to-one function:** A function whose inverse is also a function. [If  $x_1 \neq x_2$ , then  $f(x_1) \neq f(x_2)$ ]
  - \* **Horizontal-line test:** If every horizontal line intersects the graph of  $f$  in at most one point, then  $f$  is one-to-one.
  - \* The graph of a function  $f$  and its inverse  $f^{-1}$  are symmetric with respect to the line  $y = x$ .
  - \* To find the inverse,  $f^{-1}(x)$ , of a one-to-one function:
    - (1) Let  $y = f(x)$
    - (2) Interchanging the variables  $x$  and  $y$
    - (3) Solve for  $y$  and replace  $y$  by  $f^{-1}(x)$
    - (4) Check the result by showing that  $f^{-1}(f(x)) = x$  and  $f(f^{-1}(x)) = x$
  - \* To find the range of a one-to-one function  $f$ , find the domain of the inverse function  $f^{-1}$ .
- [Domain of  $f$  = Range of  $f^{-1}$ ; Range of  $f$  = Domain of  $f^{-1}$ .]

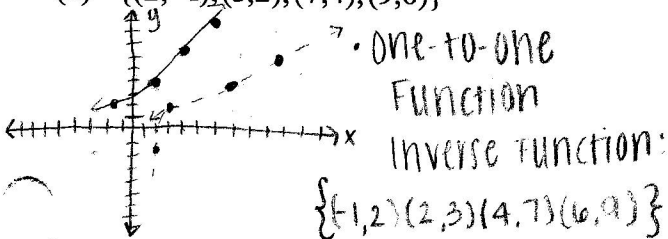
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1. Determine whether the given function is one-to-one. If it is one-to-one, find the inverse.

[Hint: Check to see if there are ordered pairs with different first coordinates and the same second coordinate. If there are, the function is not one-to-one. We can find its inverse by interchanging the x- and y-coordinates in each ordered pair.]

(a)  $\{(2,-1), (3,2), (7,4), (9,6)\}$

(b)  $\{(-2,4), (0,0), (2,4), (4,16)\}$



- NOT a one to one function  
- NO INVERSE FUNCTION.

2. Use the graph to determine whether the function is one-to-one. [Hint: use Horizontal-line test]

(a)  $f(x) = x^3$

(b)  $f(x) = x^2$

It is a ~~one-to-one~~ function, and can have inverse function.

Not a ~~one-to-one~~ function, can not have inverse function.

It is a one to one function.

Not a one to one function.

3. In the following problems, determine whether the function  $f$  is one-to-one. If it is, find the inverse of each function.

(a)  $\sqrt{x-3}$

①  $y = \sqrt{x-3}$

③  $x = \sqrt{y-3}$

④  $f^{-1}(x) = x^2 + 3$

It is a function

②  $x = \sqrt{y-3}$

$(x)^2 = (\sqrt{y-3})^2$   
 $x^2 = y-3$   
 $y = x^2 + 3$

domain:  $[3, \infty)$

inverse domain:

So neat!

range:  $[0, \infty)$

$[0, \infty)$

inverse range:

$[3, \infty)$

(b)  $\frac{3}{x-2}$

①  $y = \frac{3}{x-2}$

③  $y-2 = \frac{3}{x}$

$y = \frac{3}{x} + 2$

domain:  
 $\{x | x \neq 2\}$

②  $\frac{x}{1} = \frac{3}{y-2} \rightarrow (y-2) \frac{x}{1} = \frac{3}{1}$

④  $f^{-1}(x) = \frac{3}{x} + 2$

range:  
 $\{y | y \neq 0\}$

inverse domain:  $\{x | x \neq 0\}$  inverse range:  $\{y | y \neq 2\}$