

Useful Guidelines:

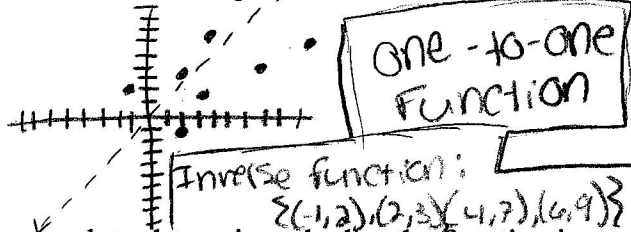
- * **One-to-one function:** A function whose inverse is also a function. [If $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$]
- * **Horizontal-line test:** If every horizontal line intersects the graph of f in at most one point, then f is one-to-one.
- * The graph of a function f and its inverse f^{-1} are symmetric with respect to the line $y = x$.
- * To find the inverse, $f^{-1}(x)$, of a one-to-one function:
 - (1) Let $y = f(x)$
 - (2) Interchanging the variables x and y
 - (3) Solve for y and replace y by $f^{-1}(x)$
 - (4) Check the result by showing that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$
- * To find the range of a one-to-one function f , find the domain of the inverse function f^{-1} .
 [Domain of f = Range of f^{-1} ; Range of f = Domain of f^{-1} .]

2/19/08

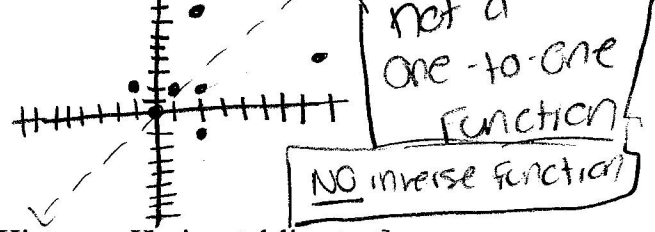
1. Determine whether the given function is one-to-one. If it is one-to-one, find the inverse.

[Hint: Check to see if there are ordered pairs with different first coordinates and the same second coordinate. If there are, the function is not one-to-one. We can find its inverse by interchanging the x- and y-coordinates in each ordered pair.]

(a) $\{(2,-1), (3,2), (7,4), (9,6)\}$

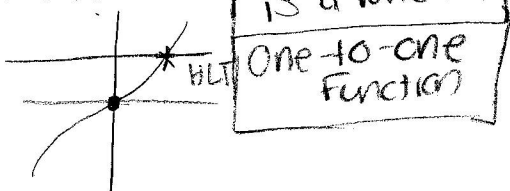


(b) $\{(-2,4), (0,0), (2,4), (4,16)\}$

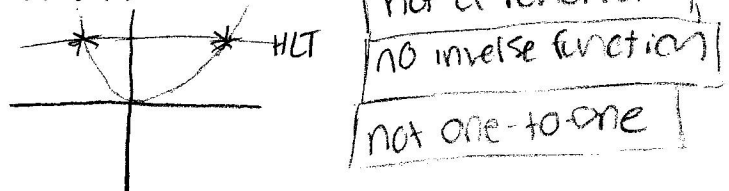


2. Use the graph to determine whether the function is one-to-one. [Hint: use Horizontal-line test]

(a) $f(x) = x^3$



(b) $f(x) = x^2$



3. In the following problems, determine whether the function f is one-to-one. If it is, find the inverse of each function.

(a) $\sqrt{x-3}$

it is a function
 d: $[3, \infty)$ inverse d: $[0, \infty)$
 r: $[0, \infty)$ inverse r: $[3, \infty)$
 One-to-one function

1) $y = \sqrt{x-3}$

2) swap x & y
 $x = \sqrt{y-3}$
 $x^2 = y-3$

3) $y = x^2 + 3$

4) $f^{-1}(x) = x^2 + 3$

(b) $\frac{3}{x-2}$

$y = \frac{3}{x-2}$

$y-2 = \frac{3}{x}$

$\frac{x}{1} = \frac{3}{y-2} \Rightarrow (y-2)x = 3$

$y = \frac{3}{x} + 2$

Inverse
 d: $\{x | x \neq 2\}$ d: $\{x | x \neq 0\}$
 r: $\{y | y \neq 0\}$ r: $\{y | y \neq 2\}$