

Useful Guidelines:

- * **One-to-one function:** A function whose inverse is also a function. [If $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$]
 - * **Horizontal-line test:** If every horizontal line intersects the graph of f in at most one point, then f is one-to-one.
 - * The graph of a function f and its inverse f^{-1} are symmetric with respect to the line $y = x$.
 - * To find the inverse, $f^{-1}(x)$, of a one-to-one function:
 - (1) Let $y = f(x)$
 - (2) Interchanging the variables x and y
 - (3) Solve for y and replace y by $f^{-1}(x)$
 - (4) Check the result by showing that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$
 - * To find the range of a one-to-one function f , find the domain of the inverse function f^{-1} .
- [Domain of f = Range of f^{-1} ; Range of f = Domain of f^{-1} .]

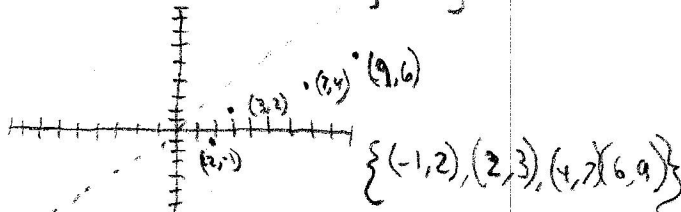
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1. Determine whether the given function is one-to-one. If it is one-to-one, find the inverse.

[Hint: Check to see if there are ordered pairs with different first coordinates and the same second coordinate. If there are, the function is not one-to-one. We can find its inverse by interchanging the x- and y-coordinates in each ordered pair.]

(a) $\{(2,-1), (3,2), (7,4), (9,6)\}$ $y = x$

(b) $\{(-2,4), (0,0), (2,4), (4,16)\}$



Not a one-to-one function
There is no inverse function

$f(x)$ = function of x

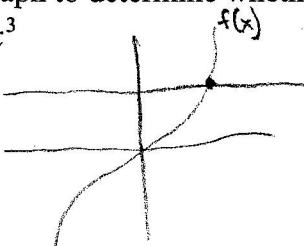
$f^{-1}(x)$ = Inverse function of $f(x)$

2. Use the graph to determine whether the function is one-to-one. [Hint: use Horizontal-line test]

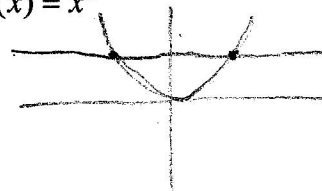
(a) $f(x) = x^3$

(b) $f(x) = x^2$

HLT



yes
one to one

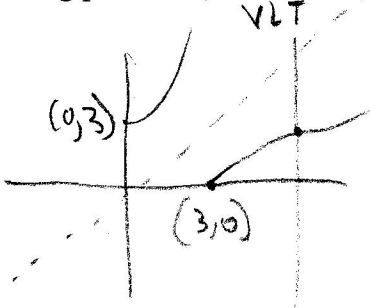


HLT

Not a one-to-one function

3. In the following problems, determine whether the function f is one-to-one. If it is, find the inverse of each function.

(a) $\sqrt{x-3}$



$$y = \sqrt{x-3}$$

$$(x)^2 = (\sqrt{y-3})^2$$

$$x^2 = y-3$$

$$y = x^2 + 3$$

$$f^{-1}(x) = x^2 + 3$$

$$D: [3, \infty)$$

$$R: [0, \infty)$$

(b) $\frac{3}{x-2}$

$$y = \frac{3}{x-2}$$

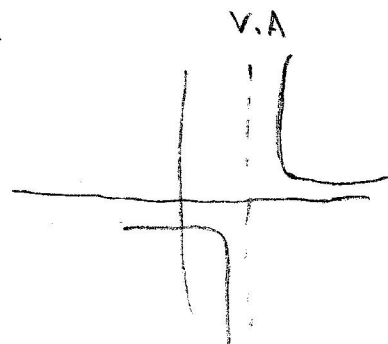
$$f^{-1}(x) = y = \frac{3}{x} + 2$$

$$x = \frac{3}{y-2}$$

$$(y-2) \cdot \frac{x}{x} = \frac{3}{x}$$

$$y-2 = \frac{3}{x}$$

$$\rightarrow y = \frac{3}{x} + 2$$



V.A

$$D: \{x | x \neq 2\}$$

$$R: \{y | y \neq 0\}$$

H.A