COLLEGE ALGEBRA

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GPS # 27 | 3.2 | LOGARITHMIC FUNCTIONS I | Class Time: 11:30-12:45 Date: 3/6/08

Useful Guidelines:

* The logarithmic function to the base a, where $a > 0$ and $a \neq 1$: $y = \log_a x$ if and only if $x = a^y$;
* Properties of the logarithmic Function $y = \log_a x$ (where $a > 0$ and $a \neq 1$):
  (1) Domain: the interval $(0, \infty)$; Range: the interval $(-\infty, \infty)$;
  (2) x-intercepts: 1; y-intercept: none;
  (3) Vertical asymptote: $x = 0$;
  (4) $f(x) = \log_a x, a > 1$, is an increasing, one-to-one, smooth and continuous function;
  $f(x) = \log_a x, 0 < a < 1$, is a decreasing, one-to-one, smooth and continuous function;
  (5) The points $(1,0), (a,1)$, and $(\frac{1}{a}, -1)$ are always on the graph of f.
* Natural logarithm function: $y = \log_e x = \ln x$ if and only if $x = e^y$.
* Common logarithm function: $y = \log x$ if and only if $x = 10^y$.

1. Change each logarithmic expression to an equivalent expression involving an exponent.

a) $\log_{10} m = 5$
$\downarrow$
$m = 10^5$

b) $\log_e b = 4$
$\downarrow$
$b = e^4$

c) $\log_p 3 = x$
$\downarrow$
$3 = p^x$

d) $\log_2 M = c$
$\downarrow$
$M = 2^c$

e) $\ln 5 = x$
$x = \log_e(5)$ $5 = e^x$

f) $\ln x = 3$
$3 = \log_e(x)$ $3 = e^x$

2. Find the exact value of the following:

a) $y = \log_3 27$
$= \log_3(3^3)$
$= 3 \log_3(3)^1 = 3 \cdot 1 = 3$

b) $y = \log_{10} 100$
$= \log_{10}(10^2)$
$= 2 \log_{10}(10)^1 = 2 \cdot 1 = 2$

c) $y = \log_3 \frac{1}{9}$
$= \log_3(3^{-2})$
$= -2 \log_3(3) = -2 \cdot 1$
$= -2$

d) $y = \log_{10} \frac{1}{1000}$
$= \log_{10}(10^{-3})$
$= -3 \log_{10}(10) = -3 \cdot 1 = -3$

e) $y = \ln e^4$
$= \log_e(e^4)$
$= 4 \log_e(e) = 4 \cdot 1 = 4$

f) $y = \ln \sqrt{e}$
$\log_e(\sqrt{e}) = \log_e(e^{1/2})$
$1/2 \log_e(e) = 1/2 \cdot 1 = 1/2$

3. Find the domain of each function.

a) $f(x) = \log_3(x+1)$
$x + 1 > 0$ d: $\{x \mid x > -1\}$
$x > -1$

b) $g(x) = 4 + 2\ln(5x)$
$5x > 0$ d: $\{x \mid x > 0\}$
$x > 0$

c) $f(x) = \sqrt{\ln x}$
$\ln x \geq 0$ d: $\{x \mid x \geq 1\}$
$x \geq e^0$

d) $g(x) = \frac{1}{\ln x}$
$x > 0$ $\ln(x) \neq 0$
$e^{\ln(x)} \neq e^0$
$\ln x \neq 0$ $x \neq 1$
d: $\{x \mid x > 0, x \neq 1\}$