

Useful Guidelines:

- * The logarithmic function to the base a , where $a > 0$ and $a \neq 1$: $y = \log_a x$ if and only if $x = a^y$;
- * Properties of the logarithmic Function $y = \log_a x$ (where $a > 0$ and $a \neq 1$):
 - (1) Domain: the interval $(0, \infty)$; Range: the interval $(-\infty, \infty)$;
 - (2) x -intercepts: 1; y -intercept: none;
 - (3) Vertical asymptote: $x = 0$;
 - (4) $f(x) = \log_a x, a > 1$, is an increasing, one-to-one, smooth and continuous function;
 $f(x) = \log_a x, 0 < a < 1$, is a decreasing, one-to-one, smooth and continuous function;
 - (5) The points $(1,0), (a,1)$, and $(\frac{1}{a}, -1)$ are always on the graph of f .
- * Natural logarithm function: $y = \log_e x = \ln x$ if and only if $x = e^y$.
- * Common logarithm function: $y = \log x$ if and only if $x = 10^y$.

20/20 Grade

$\ln = \log_e$

$\sqrt{x} = x^{\frac{1}{2}}$

find base

1. Change each logarithmic expression to an equivalent expression involving an exponent.

a) $\log_{10} m = 5$
 $\frac{\log_{10} m}{+10} = \frac{5}{+10}$

$m = 10^5$

b) $\log_e b = 4$
 $\frac{\log_e b}{+e} = \frac{4}{+e}$

$b = e^4$

c) $\log_p 3 = x$
 $\frac{\log_p 3}{+p} = \frac{x}{+p}$

$3 = p^x$

d) $\log_2 M = c$
 $\frac{\log_2 M}{+2} = \frac{c}{+2}$

$m = 2^c$

e) $\ln 5 = x$
 $\frac{\log_e(5)}{+e} = \frac{x}{+e}$

$5 = e^x$

f) $\ln x = 3$
 $\frac{\log_e(x)}{+e} = \frac{3}{+e}$

$x = e^3$

2. Find the exact value of the following:

a) $y = \log_3 27$
 $y = \log_3(3^3)$
 $= 3 \log_3(3)$
 $= 3 \cdot 1$

$= 3$

b) $y = \log_{10} 100$
 $y = \log_{10}(10^2)$
 $y = 2 \log_{10}(10)$
 $y = 2 \cdot 1$

$= 2$

c) $y = \log_3 \frac{1}{9}$
 $y = \log_3(3^{-2})$
 $y = 2 \log_3(3)$
 $y = 2 \cdot 1$

$= -2$

d) $y = \log_{10} \frac{1}{1000}$
 $y = \log_{10}(10^{-3})$
 $y = -3 \log_{10}(10)$
 $y = -3 \cdot 1$

$= -3$

e) $y = \ln(e^4)$
 $y = \log_e(e^4)$
 $y = 4 \log_e(e)$

$= 4$

f) $y = \ln(\sqrt{e})$
 $y = \log_e(e^{\frac{1}{2}})$
 $y = \frac{1}{2} \log_e(e)$
 $y = \frac{1}{2} \cdot 1$

$= \frac{1}{2}$

3. Find the domain of each function.

a) $f(x) = \log_3(x+1)$
 $x+1 > 0$
 $\frac{-1}{-1} \quad \frac{-1}{-1}$
 $D: \{x | x > -1\}$
 $x > -1$

b) $g(x) = 4 + 2 \ln(5x)$
 $D: \{x | x > 0\}$
 $\frac{5x}{5} > \frac{0}{5}$
 $x > 0$

look for input

c) $f(x) = \sqrt{\ln x}$
 $e^{\ln(x)} \geq e^0$
 $D: \{x | x \geq 1\}$
 $x \geq 1$

d) $g(x) = \frac{1}{\ln x}$
 $x > 0$
 $\ln(x) \neq 0$
 $e^{\ln(x)} \neq e^0$

$D: \{x | x > 0, x \neq 1\}$
 $x \neq 1$