

**Useful Guidelines:**

- \* The logarithmic function to the base  $a$ , where  $a > 0$  and  $a \neq 1$ :  $y = \log_a x$  if and only if  $x = a^y$ ;
- \* Properties of the logarithmic Function  $y = \log_a x$  (where  $a > 0$  and  $a \neq 1$ ):
  - (1) Domain: the interval  $(0, \infty)$ ; Range: the interval  $(-\infty, \infty)$ ;
  - (2)  $x$ -intercepts: 1;  $y$ -intercept: none;
  - (3) Vertical asymptote:  $x = 0$ ;
  - (4)  $f(x) = \log_a x, a > 1$ , is an increasing, one-to-one, smooth and continuous function;  
 $f(x) = \log_a x, 0 < a < 1$ , is a decreasing, one-to-one, smooth and continuous function;
  - (5) The points  $(1,0), (a,1)$ , and  $(\frac{1}{a}, -1)$  are always on the graph of  $f$ .
- \* Natural logarithm function:  $y = \log_e x = \ln x$  if and only if  $x = e^y$ .
- \* Common logarithm function:  $y = \log x$  if and only if  $x = 10^y$ .

*20/ Guaranteed  
no Ab.*

1. Change each logarithmic expression to an equivalent expression involving an exponent.

a)  $\log_{10} m = 5$

$m = 10^5$

b)  $\log_e b = 4$

$b = e^4$

c)  $\log_p 3 = x$

$3 = p^x$

d)  $\log_2 M = c$

$M = 2^c$

e)  $\ln 5 = x$

$\ln(x) = \log_e(x)$   
 $5 = e^x$

f)  $\ln x = 3$

$\log_e(x) = 3$   
 $x = e^3$

2. Find the exact value of the following:

a)  $y = \log_3 27$

$y = \log_3(3^3)$   
 $= 3 \log_3(3) = 3 \cdot 1 = 3$

b)  $y = \log_{10} 100$

$y = \log_{10}(10^2)$   
 $= 2 \log_{10}(10) = 2 \cdot 1 = 2$

c)  $y = \log_3 \frac{1}{9}$

$y = \log_3(3^{-2})$   
 $y = -2 \log_3(3) = -2 \cdot 1 = -2$

d)  $y = \log_{10} \frac{1}{1000}$

$y = \log_{10}(\frac{1}{10^3}) = \log_{10}(10^{-3})$   
 $= -3 \log_{10}(10) = -3$

e)  $y = \ln e^4$

$y = \log_e(e^4)$   
 $= 4 \log_e(e) = 4 \cdot 1 = 4$

f)  $y = \ln \sqrt{e}$

$= \log_e(e^{\frac{1}{2}})$   
 $= \frac{1}{2}$

3. Find the domain of each function.

a)  $f(x) = \log_3(x+1)$   $x+1 > 0$   
 $x > -1$

D:  $\{x | x > -1\}$

b)  $g(x) = 4 + 2 \ln(5x)$

D:  $\{x | x > 0\}$   $5x > 0$   
 $x > 0$

c)  $f(x) = \sqrt{\ln x}$

$\ln(x) \geq 0 \Rightarrow e^{\ln(x)} \geq e^0$   
 D:  $\{x | x \geq 1\}$   $x \geq 1$

d)  $g(x) = \frac{1}{\ln x}$   $x > 0$   
 $e^{\ln(x)} \neq 0$   
 $x \neq 1$

D:  $\{x | x > 0, x \neq 1\}$