COLLEGE ALGEBRA

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GPS # 29 **3.3** **PROPERTIES OF LOGARITHMS** Class Time: 11:30 Date: 3/11/08

Useful Guidelines:

Properties of Logarithms: $M > 0$ and $N > 0$

The logarithmic function to the base a, where $a > 0$ and $a \neq 1$: $y = \log_a x$ if and only if $x = a^y$;

The logarithmic function to the base b, where $b > 0$ and $b \neq 1$: $y = \log_b x$ if and only if $x = b^y$;

* $\log_a(1) = 0$; $\log_a(a) = 1$; $\log_a(M)^r = r\log_a(M)$ $\ln(1) = 0$; $\ln(e) = 1$; $\ln(e)^2 = 2\ln(e) = 2$; $\ln(e)^x = x$
* $a^{\log_a(M)} = M$ $e^{\ln(x)} = x$
* $\log_a(MN) = \log_a(M) + \log_a(N)$ $\ln(MN) = \ln(M) + \ln(N)$
* $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$ $\ln\left(\frac{M}{N}\right) = \ln(M) - \ln(N)$
* $\log_a(M) = \frac{\log_b(M)}{\log_b(a)}$ "Change-of-Base Formula" $\log_a(M) = \frac{\ln(M)}{\ln(a)}$
* If $M = N$, then $\log_a(M) = \log_a(N)$; If $\log_a(M) = \log_a(N)$, then $M = N$.

20/20 Good Job!

1. Write each expression as a sum and/or difference of logarithms. Express powers as factors.

a) $y = \log_3(2x\sqrt{x+4}), x > 0$

$= \log_3(2) + \log_3(x) + \log_3(\sqrt{x+4})$

$= \log_3(2) + \log_3(x) + \frac{1}{2}\log_3(x+4)$

b) $y = \ln\left(\frac{e}{\sqrt{x-1}}\right), x > 1$

$= \ln(e) - \frac{1}{2}\ln(x-1) = 1 - \frac{1}{2}\ln(x-1)$

c) $y = \log\left[\frac{\sqrt[3]{x+1}}{(x-5)^2}\right], x > 5$

$= \log\left[\frac{(x+1)^{\frac{1}{3}}}{(x-5)^2}\right] = \frac{1}{3}\log(x+1) - 2\log(x-5)$

d) $y = \ln\left[\frac{3x\sqrt{1+x^2}}{(x-2)^3}\right], x > 2$

$y = \ln\left[\frac{3x(1+x^2)^{\frac{1}{2}}}{(x-2)^3}\right]$

$= \ln(3) + \ln(x) + \frac{1}{2}\ln(1+x^2) - 3\ln(x-2)$

2. Write each expression as a single logarithm.

a) $y = \log(x) + 4\log(C)$ $= \log(x) + \log(c^4)$

$= \log(x \cdot c^4)$

b) $y = \ln(A) + \ln(e^x)$

$= \ln(A \cdot e^x) = \ln(Ae^x)$

c) $y = \log_2(x^2 - 1) + 4\log_2(x) - 7\log_2(x+1)$

$= \log_2(x^2-1) + \log_2(x^4) - \log_2[(x+1)^7]$

$= \log_2\left[\frac{(x^2-1)x^4}{(x+1)^7}\right] = \log_2\left[\frac{x^6 - x^4}{(x+1)^7}\right]$

d) $y = \ln\left(\frac{x}{x-1}\right) + \ln\left(\frac{x+1}{x}\right)$

$= \ln\left[\left(\frac{x}{x-1}\right)\left(\frac{x+1}{x}\right)\right] = \ln\left(\frac{x+1}{x-1}\right)$

3. Use the Change-of-Base Formula to evaluate each logarithm.

a) $\log_3 25$ $= \frac{\log_{10}(25)}{\log_{10}(3)}$

b) $\log_5 e$ $= \frac{\log_e(e)}{\log_e(5)} = \frac{1}{\ln(5)}$ or $\frac{1}{\log_e(5)}$

c) $\log_2 51$ $= \frac{\log_{10}(51)}{\log_{10}(2)}$

d) $\log_\pi 8$ $= \frac{\log_{10}(8)}{\log_{10}(\pi)} = \frac{\ln(8)}{\ln(\pi)}$