COLLEGE ALGEBRA

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GPS # 30 | 3.3 | **PROPERTIES OF LOGARITHMS II** | Class Time: T/Th 11:30 Date: 3-11-08

Useful Guidelines: * same base needed

Properties of Logarithms: $M > 0$ and $N > 0$

The logarithmic function to the base a, where $a > 0$ and $a \neq 1$: $y = \log_a x$ if and only if $x = a^y$;

The logarithmic function to the base b, where $b > 0$ and $b \neq 1$: $y = \log_b x$ if and only if $x = b^y$;

* $\log_a(1) = 0$; $\log_a(a) = 1$; $\log_a(M)^r = r\log_a(M)$	$\ln(1) = 0$; $\ln(e) = 1$; $\ln(e)^2 = 2\ln(e) = 2$; $\ln(e)^x = x$
* $a^{\log_a(M)} = M$	$e^{\ln(x)} = x$
* $\log_a(MN) = \log_a(M) + \log_a(N)$	$\ln(MN) = \ln(M) + \ln(N)$
* $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$	$\ln\left(\frac{M}{N}\right) = \ln(M) - \ln(N)$
* $\log_a(M) = \frac{\log_b(M)}{\log_b(a)}$ "Change-of-Base Formula"	$\log_a(M) = \frac{\ln(M)}{\ln(a)}$

* If $M = N$, then $\log_a(M) = \log_a(N)$; If $\log_a(M) = \log_a(N)$, then $M = N$.

Domain always has to be positive

$\log_3(x)$

Solve each equation and give the solution set.

1. a) $\log_5(2x) + \log_5(5) = 2$

$\log_5(10x) = 2 \Rightarrow 5^2$

$+5 \quad +5$

$\frac{10x}{10} = \frac{25}{10}$

$x = 2.5$

Sol set: $\{x \mid x = 2.5\}$

b) $\ln(x-1) - 2\ln(4) = \ln(e)^2 + \ln(1)$

$\ln(x-1) - \ln(4^2) = 2\ln(e) + \ln(1)$ (1, 0)

$\ln\left[\frac{(x-1)}{16}\right] = 2 \Rightarrow e^2$

$\frac{(x-1)}{16} = e^2$

$x - 1 = 16e^2$

$+1 \quad +1$

$x = 16e^2 + 1$

2. a) $\log_3(x) + \log_3(x-2) = 1$

$\log_3(x^2 - 2x) = 1 \Rightarrow 3^1$

$3 \quad 3$

$x^2 - 2x = 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x \neq -1$

Solution set: $\{x \mid x = 3\}$ Sol set: $\{x \mid x = 1 - e^4\}$ Sol set: $\{x \mid x = 16e^2 + 1\}$

b) $\ln(x+3) - \ln(x) = \ln(e)^4$

$\ln\left(\frac{x+3}{x}\right) = 4\ln(e)$

$e \quad e$

$\frac{x+3}{x} = e^4 \Rightarrow e^4$

multiply by -1

$x + 3 = xe^4$

$-xe^4 \quad -xe^4$

$x - xe^4 = -3$

$\frac{x(1-e^4)}{(1-e^4)} = \frac{-3}{(1-e^4)}$

$x = \frac{3}{1-e^4}$

3. a) $3^x = 10$

$\ln(3^x) = \ln(10)$

$\frac{x\ln(3)}{\ln(3)} = \frac{\ln(10)}{\ln(3)}$

$x = \frac{\ln(10)}{\ln(3)}$

b) $2^{x+2} = 5^x$

$\ln(2^{x+2}) = \ln(5^x)$

$x + 2\ln(2) = x\ln(5)$

$x\ln(2) + 2\ln(2) = x\ln(5)$

$-x\ln(2) \quad -x\ln(2)$

$\ln(4) = x\ln(5) - x\ln(2)$

$\ln(4) = x[\ln(5) - \ln(2)]$

$\frac{\ln(4)}{\ln\left(\frac{5}{2}\right)} = \frac{x\ln\left(\frac{5}{2}\right)}{\ln\left(\frac{5}{2}\right)}$

$x = \frac{\ln(4)}{\ln\left(\frac{2}{5}\right)}$

Sol set: $\left\{x \mid x = \frac{\ln(4)}{\ln\left(\frac{2}{5}\right)}\right\}$

4. Evaluate the following:

a) $k^{\log_k(k)} = k$ b) $\pi^{\log_\pi(\pi)} = \pi$ c) $e^{\ln(k)} = k$ d) $e^{\ln(\pi)} = \pi$