

Useful Guidelines:

Properties of Logarithms: $M > 0$ and $N > 0$

The logarithmic function to the base a , where $a > 0$ and $a \neq 1$: $y = \log_a x$ if and only if $x = a^y$;

The logarithmic function to the base b , where $b > 0$ and $b \neq 1$: $y = \log_b x$ if and only if $x = b^y$;

- * $\log_a(1) = 0$; $\log_a(a) = 1$; $\log_a(M)^r = r \log_a(M)$ $\ln(1) = 0$; $\ln(e) = 1$; $\ln(e)^2 = 2 \ln(e) = 2$; $\ln(e)^x = x$
- * $a^{\log_a(M)} = M$ $e^{\ln(x)} = x$
- * $\log_a(MN) = \log_a(M) + \log_a(N)$ $\ln(MN) = \ln(M) + \ln(N)$
- * $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$ $\ln\left(\frac{M}{N}\right) = \ln(M) - \ln(N)$
- * $\log_a(M) = \frac{\log_b(M)}{\log_b(a)}$ "Change-of-Base Formula" $\log_a(M) = \frac{\ln(M)}{\ln(a)}$
- * If $M = N$, then $\log_a(M) = \log_a(N)$; If $\log_a(M) = \log_a(N)$, then $M = N$.

my
no
Good
no

Solve each equation and give the solution set.

1. a) $\log_5(2x) + \log_5(5) = 2$

$\log_5(10x) = 2$ sol set: $\{x | x = 2.5\}$
 $5 \qquad \qquad \qquad 5$
 $10x = 25$
 $x = 2.5$

b) $\ln(x-1) - 2 \ln(4) = \ln(e)^2 + \ln(1)$

$= \ln(x-1) - 2 \ln 4^2$
 $= \ln\left[\frac{x-1}{16}\right] = 2$ $= \left[\frac{x-1}{16} = e^2\right] 16$
 $x-1 = 16e^2$
 $x = 16e^2 + 1$

2. a) $\log_3(x) + \log_3(x-2) = 1$

$\log_3(x^2 - 2x) = 1$ sol set: $\{x | x = 3\}$
 $3 \qquad \qquad \qquad 3$
 $x^2 - 2x = 3$
 $x^2 - 2x - 3 = 0$

b) $\ln(x+3) - \ln(x) = \ln(e)^4$

$\ln\left(\frac{x+3}{x}\right) = 4 \ln e$
 $e = 4$
 $\frac{x+3}{x} = e^4$
 $x+3 = xe^4$
 $x - xe^4 = -3$ $= \frac{x(1-e^4)}{(1-e^4)} = \frac{-3}{1-e^4}$
 $x = \frac{-3}{1-e^4} = \frac{3}{e^4-1}$

3. a) $3^x = 10$

$\ln(3^x) = \ln(10)$
 $x \ln(3) = \ln(10)$
 $\frac{x \ln(3)}{\ln(3)} = \frac{\ln(10)}{\ln(3)}$
 $x = \frac{\ln(10)}{\ln(3)}$

b) $2^{x+2} = 5^x$

$\ln(2^{x+2}) = \ln(5^x)$
 $(x+2) \ln(2) = x \ln(5)$
 $\frac{(x+2) \ln(2)}{\ln(2)} = \frac{x \ln(5)}{\ln(2)}$
 $x \ln(2) + 2 \ln(2) = x \ln(5)$
 $\ln(4) = x \ln(5) - x \ln(2)$
 $= x [\ln(5) - \ln(2)]$
 $\ln(4) = x \ln\left(\frac{5}{2}\right)$
 $\frac{\ln(4)}{\ln\left(\frac{5}{2}\right)} = \frac{x \ln\left(\frac{5}{2}\right)}{\ln\left(\frac{5}{2}\right)}$
 $\ln(4) - \ln(2)$

4. Evaluate the following:

- a) $3^{\log_3(k)} = k$
- b) $7^{\log_7(\pi)} = \pi$
- c) $e^{\ln(k)} = k$
- d) $e^{\ln(\pi)} = \pi$