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Useful Guidelines:

To Graph a Polynomial function, $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, $a_n \neq 0$

- * Step 1: Find the x -intercepts, if any (by solving the equation $f(x) = 0$), and the y -intercept, $f(0)$.
- * Step 2: Determine whether the graph crosses (when r is a zero of odd multiplicity) or touches (when r is a zero of even multiplicity) the x -axis at each x -intercept.
- * Step 3: Check the end behavior: For large $|x|$, the graph of f behaves like the graph of $f(x) = a_n x^n$.
- * Step 4: Determine the degree of $f = n$ and the maximum number of turning points on the graph of $f = n - 1$.
- * Step 5: Use the x -intercept(s) to find the intervals on which f is above the x -axis and the intervals on which f is below the x -axis. [Hint: pick a point between the zeros.]
- * Step 6: Plot the points and connect them with a smooth and continuous curve. .
[r is called a (real) **zero of f** , or **root of f** when $f(r) = 0$]

$f(x) = (x-4)(x+2)^2(x-2)$

(a) Find the x -intercepts and the y -intercept of the above polynomial function f .

x -intercept: $(-2, 0)$ $(2, 0)$ $(4, 0)$ } y -intercept: $(0, 32)$

(b) Determine whether the graph touches or crosses the x -axis at each x -intercept.

- At $x = 4$ the graph crosses because the zero has an odd mult.
- At $x = -2$ the graph touches because the zero has an even mult.
- At $x = 2$ the graph crosses because the zero has an odd mult.

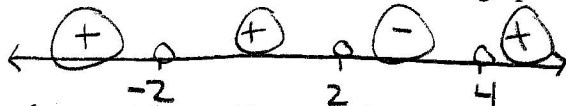
(c) Check end behavior: Find the power function that the graph of f resembles for large values of $|x|$.

It will resemble $y = x^4$ for large $|x|$

(d) Determine the maximum number of turning points of the graph of f .

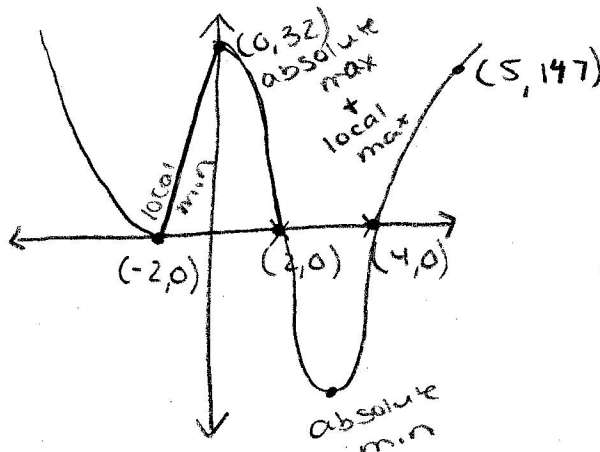
Since $n = 4$, the maximum number of turning points = $n - 1$

(e) Use the x -intercept(s) to find the intervals on which the graph of f is above and below the x -axis.



$f(-3) = (-3-4)(-3+2)^2(-3-2)$
 $f(-3) = 35$

(f) Plot the points and connect them with a smooth and continuous curve.



$f(0) = (0-4)(0+2)^2(0-2)$
 $f(0) = 32$
 $f(3) = (3-4)(3+2)^2(3-2)$
 $f(3) = -25$
 $f(5) = (5-4)(5+2)^2(5-2)$
 $f(5) = 147$