

Useful Guidelines:

To analyze the graph of a rational function, $R(x) = \frac{p(x)}{q(x)}$, in lowest terms:

- * Step 1: Find the domain of the rational function.
- * Step 2: Find the x -intercept(s), if any (let $p(x) = 0$ when $R(x)$ is in lowest term), and the y -intercept(s), $R(0)$.
- * Step 3: Write R in lowest term and find the real zeros of the denominator (vertical asymptotes).
- * Step 4: Find the horizontal or slant asymptotes, if any.
- * Step 5: Find the intervals on which R is above the x -axis and the intervals on which R is below the x -axis.
[Hint: pick a point between the zeros obtained from both the numerator and the denominator.]
- * Step 6: Graph the asymptotes, if any, plot the points, connect the points and graph R .

1. Analyze the graph of the following rational function by following Step 1 through 6 above.

$$R(x) = \frac{24}{x^2 - 4}$$

$$\frac{24}{x^2 - 4} = 0 \quad 24 \neq 0 \rightarrow x \text{ int}$$

$$1. D: \{x | x \neq \pm 2\}$$

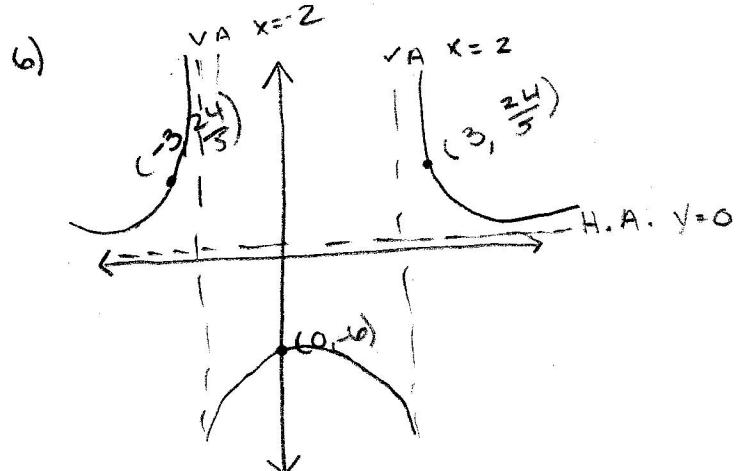
$$2. x \text{ int: none}$$

$$y \text{ int: } (0, -6)$$

$$\frac{24}{0^2 - 4} = -6 \rightarrow y \text{ int}$$

$$3. V A: x = \pm 2$$

$$R(x) = \frac{24}{x^2 - 4} = \frac{24}{(x-2)(x+2)} \quad \leftarrow \text{lowest term}$$



$$4. H.A.: y = 0$$

$$S.A.: \text{none}$$

$$5. \begin{array}{c} + \\ - \\ + \end{array}$$

$$R(-3) = \frac{24}{5}, \quad R(0) = -6, \quad R(3) = \frac{24}{5}$$

2. Solve the rational equation and give the solution set.

$$a) \frac{x}{x-5} + 1 = -3$$

$$\frac{x}{x-5} = -4 \quad (x \neq 5)$$

$$x = -4x + 20$$

$$+4x \quad +4x$$

$$\underline{5x = 20}$$

$$\text{Rev. S08} \quad x = 4$$

$$\text{So set } \{x | x = 4\}$$

$$b) \frac{16}{x^2 - 4} = 2$$

$$\frac{16}{x^2 - 4} = 2 \quad (x^2 - 4)$$

$$16 = 2x^2 - 8$$

$$+8 \quad +8$$

$$\frac{24}{2} = \frac{2x^2}{2}$$

$$\sqrt{12} = \sqrt{x^2}$$

$$\text{http://faculty.valenciac.edu/ashaw/}$$

$$x = \sqrt{12}$$

$$x = \pm 2\sqrt{3}$$

$$\begin{aligned} &\text{Solution set} \\ &\{x | x = \pm 2\sqrt{3}\} \end{aligned}$$