

Useful Guidelines:

To analyze the graph of a rational function, $R(x) = \frac{p(x)}{q(x)}$, in lowest terms:

- * Step 1: Find the domain of the rational function. \rightarrow denominator $\neq 0$
- * Step 2: Find the x -intercept(s), if any (let $p(x) = 0$ when $R(x)$ is in lowest term), and the y -intercept(s), $R(0)$.
- * Step 3: Write R in lowest term and find the real zeros of the denominator (vertical asymptotes).
- * Step 4: Find the horizontal or slant asymptotes, if any. \rightarrow only if numerator's degree \geq the denominator's
- * Step 5: Find the intervals on which R is above the x -axis and the intervals on which R is below the x -axis.
[Hint: pick a point between the zeros obtained from both the numerator and the denominator.]
- * Step 6: Graph the asymptotes, if any, plot the points, connect the points and graph R .

1. Analyze the graph of the following rational function by following Step 1 through 6 above.

$$R(x) = \frac{24}{x^2 - 4}$$

D: $\{x | x \neq \pm 2\}$

x -intercept ($y=0$) x -int = None

y -intercept ($x=0$) y -int = $(0, -6)$

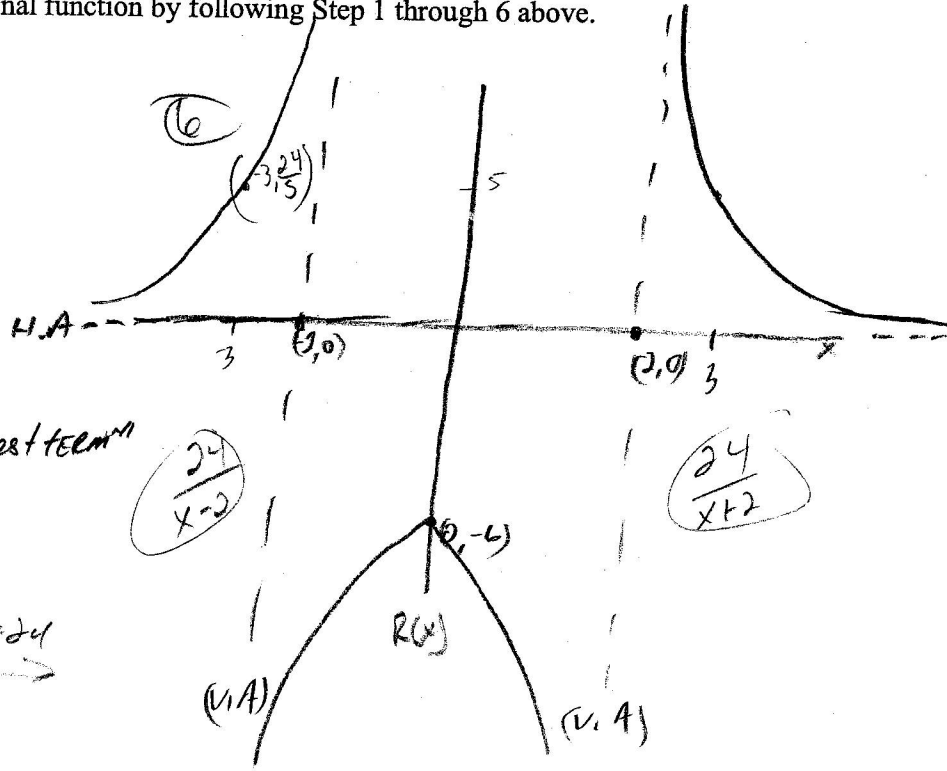
V.A. = $x = \pm 2$

$R(x) = \frac{24}{x^2 - 4} = \frac{24}{(x+2)(x-2)}$ = "lowest term"

H.A. = $(y=0)$ S.A. = NONE

$R(-3) = \frac{24}{5}$, $R(0) = -6$, $R(3) = \frac{24}{5}$

\oplus $-$ \ominus \oplus \ominus



2. Solve the rational equation and give the solution set.

a) $\frac{x}{x-5} + 1 = -3$

b) $\frac{16}{x^2 - 4} = 2$

$(x-5) \frac{x}{(x-5)} = -4(x-5)$

$(x^2 - 4) \frac{16}{x^2 - 4} = 2(x^2 - 4)$

$x = -4x + 20$
 $+4x \quad +4x$

$16 = 2(x^2 - 4)$

$\frac{5x}{5} = \frac{20}{5}$

$16 = 2x^2 - 8$
 $+8 \quad +8$

sol set $\{x | x = \pm 2\sqrt{3}\}$

Rev. S08

sol set $x = 4$
 $\{x | x = 4\}$

<http://faculty.valenciac.edu/ashaw/>

$24 = 2x^2$
 $\frac{24}{2} = \frac{2x^2}{2}$
 $12 = x^2$
 $\sqrt{12} = \sqrt{x^2}$
 $(4 \cdot 3)$

$\pm 2\sqrt{3} = x$