

MAC 1105
Module 3
System of Equations and
Inequalities

Learning Objectives

Upon completing this module, you should be able to:

1. Evaluate functions of two variables.
2. Apply the method of substitution.
3. Apply the elimination method.
4. Solve system of equations symbolically.
5. Apply graphical and numerical methods to system of equations.
6. Recognize different types of linear systems.

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Learning Objectives (Cont.)

7. Use basic terminology related to inequalities.
8. Use interval notation.
9. Solve linear inequalities symbolically.
10. Solve linear inequalities graphically and numerically.
11. Solve double inequalities.
12. Graph a system of linear inequalities.

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System of Equations and Inequalities

There are two major topics in this module:

- System of Linear Equations in Two Variables
- Solutions of Linear Inequalities

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Do We Really Use Functions of Two Variables?

- ◆ The answer is YES.
- ◆ Many quantities in everyday life depend on more than one variable.

Examples

- ◆ Area of a rectangle requires both width and length.
- ◆ Heat index is the function of temperature and humidity.
- ◆ Wind chill is determined by calculating the temperature and wind speed.
- ◆ Grade point average is computed using grades and credit hours.

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Let's Take a Look at the Arithmetic Operations

- ◆ The arithmetic operations of addition, subtraction, multiplication, and division are computed by *functions of two inputs*.
- ◆ The addition function of f can be represented symbolically by $f(x,y) = x + y$, where $z = f(x,y)$.
 - ◆ The *independent variables* are x and y .
 - ◆ The *dependent variable* is z . The z output depends on the inputs x and y .

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Here are Some Examples

For each function, evaluate the expression and interpret the result.

- a) $f(5, -2)$ where $f(x, y) = xy$
- b) $A(6, 9)$, where $A(b, h) = \frac{1}{2}bh$ calculates the area of a triangle with a base of 6 inches and a height of 9 inches.

Solution

- $f(5, -2) = (5)(-2) = -10$.
 - $A(6, 9) = \frac{1}{2}(6)(9) = 27$
- If a triangle has a base of 6 inches and a height of 9 inches, the area of the triangle is 27 square inches.

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What is a System of Linear Equations?

- ◆ A linear equation in two variables can be written in the form $ax + by = k$, where a , b , and k are constants, and a and b are not equal to 0.
- ◆ A pair of equations is called a system of linear equations because they involve solving more than one linear equation at once.
- ◆ A solution to a system of equations consists of an x -value and a y -value that satisfy both equations simultaneously.
- ◆ The set of all solutions is called the solution set.

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How to Use the Method of Substitution to solve a system of two equations?

THE METHOD OF SUBSTITUTION

To use the method of substitution to solve a system of two equations in two variables, perform the following steps.

- STEP 1:** Choose a variable in one of the two equations. Solve the equation for that variable.
- STEP 2:** Substitute the result from **STEP 1** into the other equation and solve for the remaining variable.
- STEP 3:** Use the value of the variable from **STEP 2** to determine the value of the other variable. To do this, you may want to use the equation you found in **STEP 1**.

Note: To check your answer, substitute the value of each variable into the given equations. These values should satisfy both equations.

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How to Solve the System Symbolically?

Solve the system symbolically.

$$4x + 2y = 8$$

$$3x - 7y = -11$$

Solution

Step 1: Solve one of the equations for one of the variables.

$$4x + 2y = 8$$

$$2y = -4x + 8$$

$$y = -2x + 4$$

Step 2: Substitute $-2x + 4$ for y in the second equation.

$$3x - 7(-2x + 4) = -11$$

$$3x + 14x - 28 = -11$$

$$x = 1$$

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How to Solve the System Symbolically? (Cont.)

Step 3: Substitute $x = 1$ into the equation $y = -2x + 4$ from **Step 1**. We find that

$$y = 2.$$

Check:

- $3(1) - 7(2) = -11$ $4(1) + 2(2) = 8$
- The **ordered pair** is $(1, 2)$ since the **solutions** check in **both** equations.

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Example with Infinitely Many Solutions

Solve the system.

$$8x - 2y = -4$$

• **Solution**

$$-4x + y = 2$$

- Solve the second equation for y .

$$-4x + y = 2$$

$$y = 4x + 2$$

- Substitute $4x + 2$ for y in the first equation, solving for x .

$$8x - 2(4x + 2) = -4$$

$$8x - 8x - 4 = -4$$

$$-4 = -4$$

- The equation $-4 = -4$ is an **identity** that is **always true** and indicates that **there are infinitely many solutions**. The two equations are **equivalent**.

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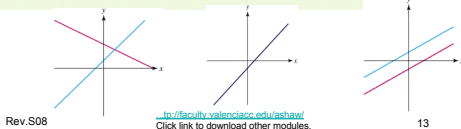
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Possible Graphs of a System of Two Linear Equations in Two Variables

POSSIBLE GRAPHS OF A SYSTEM OF TWO LINEAR EQUATIONS IN TWO VARIABLES

1. The graphs of the two equations are distinct lines that intersect at one point. The system is *consistent*. There is one solution, which is given by the coordinates of the point of intersection. In this case the equations are *independent*.
2. The graphs of the two equations are the same line. The system is *consistent*. There are infinitely many solutions, and the equations are *dependent*.
3. The graphs of the two equations are distinct parallel lines. The system is *inconsistent*. There are no solutions.



How to Use Elimination Method to Solve System of Equations?

Use **elimination** to solve each **system of equations**, if possible. Identify the system as **consistent** or **inconsistent**. If the system is consistent, state whether the equations are **dependent** or **independent**. Support your results graphically.

a) $3x - y = 7$ b) $5x - y = 8$ c) $x - y = 5$
 $5x + y = 9$ $-5x + y = -8$ $x - y = -2$

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How to Use Elimination Method to Solve System of Equations? (Cont.)

Solution

a) $3x - y = 7$
 $5x + y = 9$

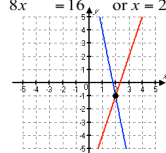
Eliminate y by adding the equations.

$$\begin{array}{r} 3x - y = 7 \\ 5x + y = 9 \\ \hline 8x = 16 \end{array}$$

$$8x = 16 \text{ or } x = 2$$

Find y by substituting $x = 2$ in either equation.

$$\begin{array}{r} 3x - y = 7 \\ 3(2) - y = 7 \\ -y = 1 \\ y = -1 \end{array}$$



The solution is $(2, -1)$. The system is **consistent** and the equations are **independent**.

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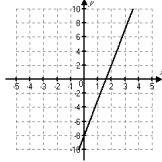
How to Use Elimination Method to Solve System of Equations? (Cont.)

b) $5x - y = 8$
 $-5x + y = -8$

If we add the equations we obtain the following result.

$$\begin{array}{r} 5x - y = 8 \\ -5x + y = -8 \\ \hline 0 = 0 \end{array}$$

The equation $0 = 0$ is an identity that is always true. The two equations are equivalent. There are infinitely many solutions. $\{(x, y) \mid 5x - y = 8\}$



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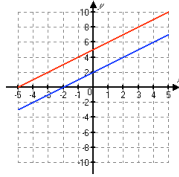
How to Use Elimination Method to Solve System of Equations? (Cont.)

c) $x - y = 5$
 $x - y = -2$

If we subtract the second equation from the first, we obtain the following result.

$$\begin{array}{r} x - y = 5 \\ -x + y = -2 \\ \hline 0 = 7 \end{array}$$

The equation $0 = 7$ is a contradiction that is never true. Therefore there are no solutions, and the system is inconsistent.



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Let's Practice Using Elimination

Solve the system by using elimination.

$3x - 4y = 1$

$2x + 3y = 12$

Solution

Multiply the first equation by 3 and the second equation by 4. Addition eliminates the y-variable.

$9x - 12y = 3$

$8x + 12y = 48$

$17x = 51 \text{ or } x = 3$

Substituting $x = 3$ in $2x + 3y = 12$ results in

$2(3) + 3y = 12 \text{ or } y = 2$

The solution is $(3, 2)$.

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Terminology related to Inequalities

- **Inequalities** result whenever the **equals sign** in an equation is **replaced with** any one of the symbols: \leq , \geq , $<$, $>$
- Examples of inequalities include:
 - $2x - 7 > x + 13$
 - $x^2 \leq 15 - 21x$
 - $xy + 9x < 2x^2$
 - $35 > 6$

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Linear Inequality in One Variable

• A **linear inequality in one variable** is an inequality that can be written in the form

$$ax + b > 0 \text{ where } a \neq 0.$$

(The symbol may be replaced by \leq , \geq , $<$, $>$)

• Examples of linear inequalities in one variable:

- $5x + 4 \leq 2 + 3x$ simplifies to $2x + 2 \leq 0$
- $-1(x - 3) + 4(2x + 1) > 5$ simplifies to $7x + 2 > 0$

• Examples of inequalities in one variable which are **not** linear:

- $x^2 < 1$

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Let's Look at Interval Notation

The solution to a **linear inequality in one variable** is typically an **interval on the real number line**. See examples of interval notation below.

TABLE 2.12 Interval Notation

Inequality	Interval Notation	Graph
$-2 < x < 2$	$(-2, 2)$ open interval	
$-1 < x \leq 3$	$(-1, 3]$ half-open interval	
$-3 \leq x \leq 2$	$[-3, 2]$ closed interval	
$x > -3$	$(-3, \infty)$ infinite interval	
$x \leq 1$	$(-\infty, 1]$ infinite interval	
$-\infty < x < \infty$ (entire number line)	$(-\infty, \infty)$ infinite interval	

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Multiplied by a Negative Number

Note that $3 < 5$, but if both sides are multiplied by -1 , in order to produce a true statement the $>$ symbol must be used.

$$3 < 5$$

but

$$-3 > -5$$

So when both sides of an inequality are multiplied (or divided) by a negative number the direction of the inequality must be reversed.

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How to Solve Linear Inequalities Symbolically?

The procedure for solving a linear inequality symbolically is the same as the procedure for solving a linear equation, **except** when both sides of an inequality are multiplied (or divided) by a negative number the direction of the inequality is reversed.

Example of Solving a
Linear Equation Symbolically

$$\begin{aligned} \text{Solve } -2x + 1 &= x - 2 \\ -2x - x &= -2 - 1 \\ -3x &= -3 \\ x &= 1 \end{aligned}$$

Example of Solving a
Linear Inequality Symbolically

$$\begin{aligned} \text{Solve } -2x + 1 &< x - 2 \\ -2x - x &< -2 - 1 \\ -3x &< -3 \\ x &> 1 \end{aligned}$$

Note that we divided both sides by -3 so the direction of the inequality was reversed. In interval notation the solution set is $(1, \infty)$.

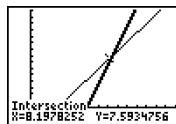
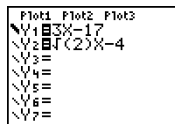
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How to Solve a Linear Inequality Graphically?

Solve $3x - 17 > \sqrt{2}x - 4$



Note that the graphs intersect at the point $(8.20, 7.59)$. The graph of y_1 is above the graph of y_2 to the right of the point of intersection or when $x > 8.20$. Thus, in interval notation, the solution set is $(8.20, \infty)$

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How to Solve Double Inequalities? (Cont.)

$$30 < 88 - 32x < 40$$

$$30 - 88 < -32x < 40 - 88$$

$$-58 < -32x < -48$$

$$\frac{-58}{-32} > x > \frac{-48}{-32}$$

$$\frac{29}{16} > x > \frac{3}{2}$$

$$1.8125 > x > 1.5$$

$$1.5 < x < 1.8125$$

Direction reversed –Divided each side of an inequality by a negative

Thus, between 1.5 and 1.8125 miles above ground level, the air temperature is between 30 and 40 degrees Fahrenheit.

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How to Graph a System of Linear Inequalities?

The graph of a **linear inequality** is a **half-plane**, which may include the boundary. The **boundary line** is included when the inequality **includes a less than or equal to or greater than or equal to symbol**.

To determine which part of the plane to shade, select a **test point**.

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How to Graph a System of Linear Inequalities? (Cont.)

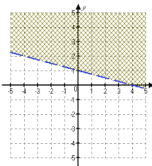
Graph the solution set to the inequality $x + 4y > 4$.

Solution

Graph the line $x + 4y = 4$ using a dashed line.

Use a **test point** to **determine which half of the plane to shade**.

Test Point	$x + 4y > 4$	True or False?
(4, 2)	$4 + 4(2) > 4$	True
(0, 0)	$0 + 4(0) > 4$	False



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Example

Solve the system of inequalities by shading the solution set. Use the graph to identify one solution.

$$x + y \leq 3$$

$$2x + y \geq 4$$

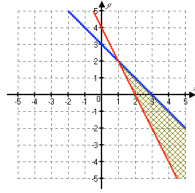
Solution

Solve each inequality for y .

$$y \leq -x + 3 \text{ (shade below line)}$$

$$y \geq -2x + 4 \text{ (shade above line)}$$

The point $(4, -2)$ is a solution.



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What have we learned?

We have learned to:

1. Evaluate functions of two variables.
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What have we learned? (Cont.)

7. Use basic terminology related to inequalities.
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Credit

Some of these slides have been adapted/modified in part/whole from the slides of the following textbook:

- Rockswold, Gary, Precalculus with Modeling and Visualization, 3th Edition

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