MAC 1140

Module 3
Linear Functions and Equations
Part II

Learning Objectives

Upon completing this module, you should be able to

1. understand basic terminology related to inequalities.
2. use interval notation.
3. solve linear inequalities symbolically.
4. solve linear inequalities graphically and numerically.
5. solve compound inequalities.
6. evaluate and graph piecewise-defined functions.
7. evaluate and graph the greatest integer function.
8. evaluate and graph the absolute value function.
9. solve absolute value equations and inequalities.

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Linear Functions and Equations

There are two sections in this module:

2.3 Linear Inequalities
2.5 Piecewise-Defined Functions

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Let’s get started by looking at the terminology related to Inequalities.

Terminology Related to Inequalities

- Inequalities result whenever the equals sign in an equation is replaced with any one of the symbols: $\le$, $\ge$, $<$, $>$
- Examples of inequalities include:
  - $2x - 7 > x + 13$
  - $x^2 \le 15 - 21x$
  - $xy + 9x < 2x^2$
  - $35 > 6$

Linear Inequality in One Variable

- A linear inequality in one variable is an inequality that can be written in the form

  $ax + b > 0$ where $a \neq 0$.

  (The symbol may be replaced by $\le$, $\ge$, $<$, $>$)

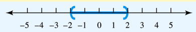
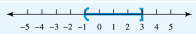
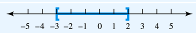
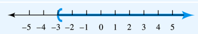
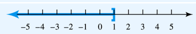
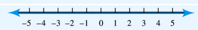
- Examples of linear inequalities in one variable:
  - $5x + 4 \le 2 + 3x$ simplifies to $2x + 2 \le 0$
  - $-1(x - 3) + 4(2x + 1) > 5$ simplifies to $7x + 2 > 0$

- Examples of inequalities in one variable which are not linear:
  - $x^2 < 1$

Let's Look at Interval Notation

- The solution to a linear inequality in one variable is typically an interval on the real number line. See examples of interval notation below.

TABLE 2.12 Interval Notation

Inequality	Interval Notation	Graph
$-2 < x < 2$	$(-2, 2)$ open interval	
$-1 < x \leq 3$	$(-1, 3]$ half-open interval	
$-3 \leq x \leq 2$	$[-3, 2]$ closed interval	
$x > -3$	$(-3, \infty)$ infinite interval	
$x \leq 1$	$(-\infty, 1]$ infinite interval	
$-\infty < x < \infty$ (entire number line)	$(-\infty, \infty)$ infinite interval	

Multiplied by a Negative Number

- Note that 3 < 5, but if both sides are multiplied by - 1, in order to produce a true statement the > symbol must be used.

3 < 5

but

- 3 > - 5

So when both sides of an inequality are multiplied (or divided) by a negative number the direction of the inequality must be reversed.

How to Solve Linear Inequalities Symbolically?

- The procedure for solving a linear inequality symbolically is the same as the procedure for solving a linear equation, except when both sides of an inequality are multiplied (or divided) by a negative number the direction of the inequality is reversed.

Example of Solving a
Linear **Equation** Symbolically

Solve $-2x + 1 = x - 2$

$-2x - x = -2 - 1$

$-3x = -3$

$x = 1$

Example of Solving a
Linear **Inequality** Symbolicly

Solve $-2x + 1 < x - 2$

$-2x - x < -2 - 1$

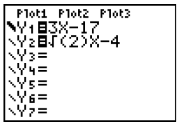
$-3x < -3$

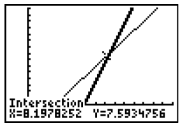
$x > 1$

Note that we divided both sides by -3 so the direction of the inequality was reversed. In interval notation the solution set is $(1, \infty)$

How to Solve a Linear Inequality Graphically?

- Solve $3x - 17 > \sqrt{2}x - 4$



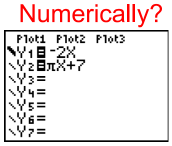


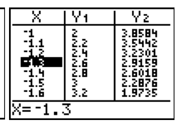
Note that the graphs intersect at the point (8.20, 7.59). The graph of y_1 is above the graph of y_2 to the right of the point of intersection or when $x > 8.20$. Thus, in interval notation, the solution set is (8.20, ∞)

How to Solve a Linear Inequality Numerically?

- Solve

$$-2x \geq \pi x + 7$$





X	Y1	Y2
-1	2	3.8584
-1.1	2.2	3.5442
-1.2	2.4	3.2301
-1.3	2.6	2.9159
-1.4	2.8	2.6018
-1.5	3	2.2876
-1.6	3.2	1.9735

X=-1.3

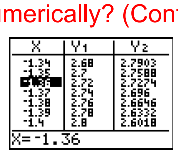
Note that the inequality above becomes $y_1 \geq y_2$ since we let y_1 equal the left-hand side and y_2 equal the right hand side.

To write the solution set of the inequality we are looking for the values of x in the table for which y_1 is the same or larger than y_2. Note that when $x = -1.3$, y_1 is less than y_2; but when $x = -1.4$, y_1 is larger than y_2. By the Intermediate Value Property, there is a value of x between - 1.4 and - 1.3 such that $y_1 = y_2$. In order to find an approximation of this value, make a new table in which x is incremented by .01 (note that x is incremented by .1 in the table to the left here.)

How to Solve a Linear Inequality Numerically? (Cont.)

- Solve

$$-2x \geq \pi x + 7$$



X	Y1	Y2
-1.34	2.68	2.7903
-1.35	2.7	2.7588
-1.36	2.72	2.7274
-1.37	2.74	2.696
-1.38	2.76	2.6646
-1.39	2.78	2.6332
-1.4	2.8	2.6018

X=-1.36

To write the solution set of the inequality we are looking for the values of x in the table for which y_1 is the same as or larger than y_2. Note that when x is approximately -1.36, y_1 equals y_2 and when x is smaller than -1.36 y_1 is larger than y_2, so the solutions can be written

$x \leq -1.36$ or (-∞, -1.36] in interval notation.

How to Solve Compound Inequalities?

- Example: Suppose the Fahrenheit temperature x miles above the ground level is given by $T(x) = 88 – 32\ x$. Determine the altitudes where the air temp is from 30^0 to 40^0.
- We must solve the inequality $30 < 88 – 32\ x < 40$

To solve: Isolate the variable x in the middle of the three-part inequality

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How to Solve Compound Inequalities? (Cont.)

$$30 < 88 - 32x < 40$$
$$30 - 88 < -32x < 40 - 88$$
$$-58 < -32x < -48$$
$$\frac{-58}{-32} > x > \frac{-48}{-32}$$

Direction reversed –Divided each side of an inequality by a negative

$$\frac{29}{16} > x > \frac{3}{2}$$
$$1.8125 > x > 1.5$$
$$1.5 < x < 1.8125$$

Thus, between 1.5 and 1.8215 miles above ground level, the air temperature is between 30 and 40 degrees Fahrenheit.

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What is a Piecewise-Defined Function?

A piecewise-defined function is simply a function defined by more than one formula on its domain.

Examples:
- Step function
- Greatest integer function
- Absolute value function

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Example of a Piecewise-Defined Function

- Undergraduate Classification at Study-Hard University (SHU) is a function of Hours Earned. We can write this in function notation as $C = f(H)$.
  - From Catalogue – Verbal Representation

```
No student may be classified as a sophomore until
after earning at least 30 semester hours.

No student may be classified as a junior until
after earning at least 60 hours.

No student may be classified as a senior until
after earning at least 90 hours.
```


Example of a Piecewise-Defined Function (Cont.)

Symbolic Representation

$$C = f(H) = \begin{cases} \text{Freshman if } 0 \le H < 30 \\ \text{Sophomore if } 30 \le H < 60 \\ \text{Junior if } 60 \le H < 90 \\ \text{Senior if } 90 \le H < 120 \end{cases}$$

- Evaluate $f(20)$
  - $f(20)$ = Freshman
- Evaluate $f(30)$
  - $f(30)$ = Sophomore
- Evaluate $f(59)$
  - $f(59)$ = Sophomore
- Evaluate $f(61)$
  - $f(61)$ = Junior
- Evaluate $f(100)$
  - $f(100)$ = Senior

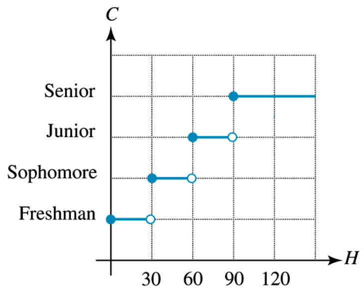
Example of a Piecewise-Defined Function (Cont.)

Symbolic Representation

$$C = f(H) = \begin{cases} \text{Freshman if } 0 \le H < 30 \\ \text{Sophomore if } 30 \le H < 60 \\ \text{Junior if } 60 \le H < 90 \\ \text{Senior if } 90 \le H < 120 \end{cases}$$

- Why is $f(20)$ = Freshman?
  - To evaluate $f(20)$ one must ask: when H has a value of 20, what is the value of C? In other words, what is the classification of a student who has earned 20 credit hours? 20 fits into the category $0 \le H < 30$, so the answer is Freshman.

The Graphical Representation

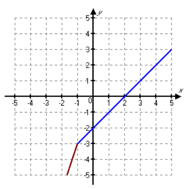


Another Example of a Piecewise-Defined Function

- Graph the following function and evaluate $f(-5)$, $f(-1)$, $f(0)$

$$f(x) = \begin{cases} 3x & \text{if } x < -1 \\ x-2 & \text{if } x \geq -1 \end{cases}$$

  - First graph the line $y = 3x$, but restrict the graph to points which have an x-coordinate < -1.
  - Now graph $x - 2$ but restrict the graph to points which have an x-coordinate -1 and larger.
  - The resulting graph is:



Another Example of a Piecewise-Defined Function (Cont.)

- Graph the following function and evaluate $f(-5)$, $f(-1)$, $f(0)$

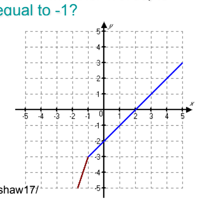
$$f(x) = \begin{cases} 3x & \text{if } x < -1 \\ x-2 & \text{if } x \geq -1 \end{cases}$$

- Given an input, to see which rule to use to compute the output, check to see into which category the input fits. In other words, is an input less than -1 or greater than or equal to -1?

- Since -5 < -1, use the rule $y = 3x$.
Thus $f(-5) = 3(-5) = -15$

- Since $-1 \geq -1$, use the rule $y = x - 2$.
Thus $f(-1) = -1 - 2 = -3$

- Since $0 \geq -1$, use the rule $y = x - 2$.
Thus $f(0) = 0 - 2 = -2$



What is the Greatest Integer Function?

- The symbol for the greatest integer less than or equal to x is $[\![x]\!]$
- The greatest integer function is a piecewise-defined function.

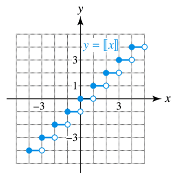


FIGURE 2.60 The Greatest Integer Function

$[\![2]\!] = 2$

$[\![2.1]\!] = 2$

$[\![2.999999]\!] = 2$

$[\![-2]\!] = -2$

$[\![-2.1]\!] = -3$

$[\![-1.9]\!] = -2$

What is the Absolute Value Function?

- The symbol for the absolute value of x is $|x|$
- The absolute value function is a piecewise-defined function.
- The output from the absolute value function is never negative.

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

$|-5| = 5$

$|-2.3| = 2$

$|0| = 0$

$|5| = 5$

$|2.3| = 2.3$

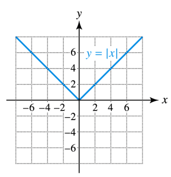


FIGURE 2.63 The Absolute Value Function

How to Solve an Absolute Value Equation?

Let k be a positive number. Then

$$|ax + b| = k$$

is equivalent to

$$ax + b = \pm k$$

- Example: Solve $|1 - 2x| = 3$

$$1 - 2x = 3 \text{ or } 1 - 2x = -3$$
$$-2x = 3 - 1 \text{ or } -2x = -3 - 1$$
$$-2x = 2 \text{ or } -2x = -4$$
$$x = -1 \text{ or } x = 2$$

How to Solve Absolute Value Inequalities?

ABSOLUTE VALUE INEQUALITIES

Let the solutions to $|ax + b| = k$ be s_1 and s_2, where $s_1 < s_2$ and $k > 0$.

1. $|ax + b| < k$ is equivalent to $s_1 < x < s_2$.
2. $|ax + b| > k$ is equivalent to $x < s_1$ or $x > s_2$.

Similar statements can be made for inequalities involving $\leq$ or $\geq$.

- Example: Solve $|1 - 2x| > 3$
  From the previous example the solutions of the equation $|1 - 2x| = 3$ are -1 and 2.
  - Thus the solutions for the inequality $|1 - 2x| > 3$ are $x < -1$ or $x > 2$.
  - In interval notation this is $(-\infty, -1) \cup (2, \infty)$

What Have We learned?

We have learned to

1. understand basic terminology related to inequalities.
2. use interval notation.
3. solve linear inequalities symbolically.
4. solve linear inequalities graphically and numerically.
5. solve compound inequalities.
6. evaluate and graph piecewise-defined functions.
7. evaluate and graph the greatest integer function.
8. evaluate and graph the absolute value function.
9. solve absolute value equations and inequalities.

Credit

Some of these slides have been adapted/modified in part/whole from the slides of the following textbook:

- Rockswold, Gary, Precalculus with Modeling and Visualization, 4th Edition
- Rockswold, Gary, Precalculus with Modeling and Visualization, 5th Edition