MAC 1114

Module 2
Acute Angles and Right Triangles

Learning Objectives

Upon completing this module, you should be able to:

1. Express the trigonometric ratios in terms of the sides of the triangle given a right triangle.
2. Apply right triangle trigonometry to find function values of an acute angle.
3. Solve equations using the cofunction identities.
4. Find trigonometric function values of special angles.
5. Find reference angles.
6. Find trigonometric function values of non-acute angles using reference angles.
7. Evaluate an expression with function values of special angles.

Learning Objectives (Cont.)

8. Use coterminal angles to find function values .
9. Find angle measures given an interval and a function value.
10. Find function values with a calculator.
11. Use inverse trigonometric functions to find angles.
12. Solve a right triangle given an angle and a side.
13. Solve a right triangle given two sides.
14. Solve applied trigonometry problems.

Acute Angles and Right Triangles

There are four major topics in this module:

- Trigonometric Functions of Acute Angles
- Trigonometric Functions of Non-Acute Angles
- Finding Trigonometric Function Values Using a Calculator
- Solving Right Triangles

What are the Right-Triangle Based Definitions of Trigonometric Functions?

- For any acute angle A in standard position.

$$\sin A = \frac{y}{r} = \frac{\text{side opposite}}{\text{hypotenuse}} \qquad \csc A = \frac{r}{y} = \frac{\text{hypotenuse}}{\text{side opposite}}$$

$$\cos A = \frac{x}{r} = \frac{\text{side adjacent}}{\text{hypotenuse}} \qquad \sec A = \frac{r}{x} = \frac{\text{hypotenuse}}{\text{side adjacent}}$$

$$\tan A = \frac{y}{x} = \frac{\text{side opposite}}{\text{side adjacent}}$$

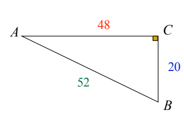
$$\cot A = \frac{x}{y} = \frac{\text{side adjacent}}{\text{side opposite}}.$$

Tip: Use the mnemonic sohcahtoa to remember that "sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, and tangent is opposite over adjacent."

Example of Finding Function Values of an Acute Angle

- Find the values of sin A, cos A, and tan A in the right triangle shown.

- $$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{20}{52}$$
  $$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{48}{52}$$
  $$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{20}{48}$$



Cofunction Identities

- For any acute angle A,
- $\sin A = \cos(90° - A)$ $\csc A = \sec(90° - A)$
- $\tan A = \cot(90° - A)$ $\cos A = \sin(90° - A)$
- $\sec A = \csc(90° - A)$ $\cot A = \tan(90° - A)$

Example of Writing Functions in Terms of Cofunctions

- Write each function in terms of its cofunction.

- a) cos 38°

$\cos 38° = \sin(90° - 38°)$
$= \sin 52°$

- b) sec 78°

$\sec 78° = \csc(90° - 78°)$
$= \csc 12°$

Example of Solving Trigonometric Equations Using the Cofunction Identities

- Find one solution for the equation $\cot(4\theta + 8°) = \tan(2\theta + 4°)$.

Assume all angles are acute angles.

$$\cot(4\theta + 8°) = \tan(2\theta + 4°)$$
$$(4\theta + 8°) + (2\theta + 4°) = 90°$$

This is due to tangent and cotangent are cofunctions.

$$6\theta + 12° = 90°$$
$$6\theta = 78°$$
$$\theta = 13°$$

Example of Comparing Function Values of Acute Angles

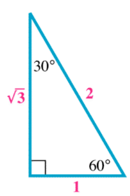
- Tell whether the statement is *true* or *false*.

  sin 31° > sin 29°

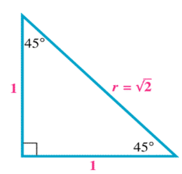
- In the interval from 0° to 90°, as the angle increases, so does the sine of the angle, which makes sin 31° > sin 29° a true statement.

Two Special Triangles

- 30-60-90 Triangle



- 45-45-90 Triangle



45°– 45° right triangle

Can you reproduce these two triangles without looking at them? Try it now. It would be very handy for you later.

Function Values of Special Angles

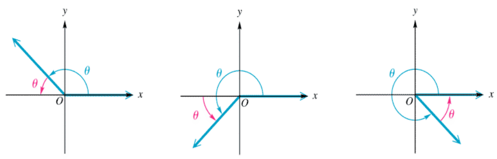
Remember the mnemonic sohcahtoa - "sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, and tangent is opposite over adjacent."

θ	sin θ	cos θ	tan θ	cot θ	sec θ	csc θ
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$

Now, try to use your two special triangles to check out these function values.

What is a Reference Angle?

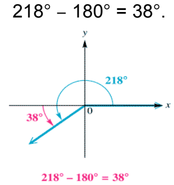
- A reference angle for an angle θ is the positive acute angle made by the terminal side of angle θ and the *x*-axis.



θ in quadrant II   θ in quadrant III   θ in quadrant IV

Example of Finding the Reference Angle for Each Angle

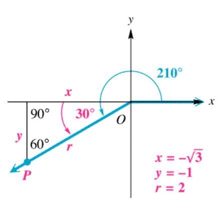
- a) 218°
- Positive acute angle made by the terminal side of the angle and the *x*-axis is
  218° – 180° = 38°.



- 1387°
- Divide 1387 by 360 to get a quotient of about 3.9. Begin by subtracting 360 three times.
  1387° – 3(360°) = 307°.
- The reference angle for 307 ° is 360° – 307 ° = 53°

How to Find Trigonometric Function Values of a Quadrant Angle?

- Find the values of the trigonometric functions for 210°.
- Reference angle:
  210° – 180° = 30°
  Choose point *P* on the terminal side of the angle so the distance from the origin to *P* is 2.



How to Find Trigonometric Function Values of a Quadrant Angle (cont.)

- The coordinates of *P* are $\left(-\sqrt{3},-1\right)$
  - *x* = $-\sqrt{3}$ *y* = –1 *r* = 2

$$\sin 210^\circ = -\frac{1}{2} \qquad \cos 210^\circ = -\frac{\sqrt{3}}{2} \qquad \tan 210^\circ = \frac{\sqrt{3}}{3}$$

$$\csc 210^\circ = -2 \qquad \sec 210^\circ = -\frac{2\sqrt{3}}{3} \qquad \cot 210^\circ = \sqrt{3}$$

Tip: Use the mnemonic cast - “cosine, all*, sine, tangent” for positive sign in the four quadrants - start from the fourth quadrant and go counterclockwise. Alternatively, use the table of signs on page 28 in section 1.4. (Note all* will include sine, cosine and tangent.)

How to Find Trigonometric Function Values for Any Nonquadrantal angle?

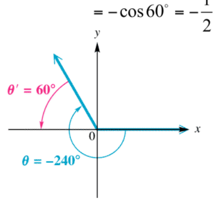
- *Step* 1 If θ > 360°, or if θ < 0°, then find a coterminal angle by adding or subtracting 360° as many times as needed to get an angle greater than 0° but less than 360°.
- *Step* 2 Find the reference angle θ*′*.
- *Step* 3 Find the trigonometric function values for reference angle θ*′*.
- *Step* 4 Determine the correct signs for the values found in Step 3. (Use the mnemonic cast or use the table of signs in section 1.4, if necessary.) This gives the values of the trigonometric functions for angle θ.

Example of Finding Trigonometric Function Values Using Reference Angles

- Find the exact value of each expression.
- cos (–240°)
- Since an angle of –240° is coterminal with an angle of –240° + 360° = 120°, the reference angles is 180° – 120° = 60°, as shown.

$$\cos(-240^\circ) = \cos 120^\circ$$

$$= -\cos 60^\circ = -\frac{1}{2}$$



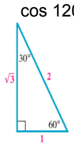
How to Evaluate an Expression with Function Values of Special Angles?

- Evaluate cos 120° + 2 sin² 60° – tan² 30°.
- Since $\cos 120^\circ = -\frac{1}{2}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$, and $\tan 30^\circ = \frac{\sqrt{3}}{3}$,

$$\cos 120^\circ + 2\sin^2 60^\circ - \tan^2 30^\circ = -\frac{1}{2} + 2\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{3}\right)^2$$

$$= -\frac{1}{2} + 2\left(\frac{3}{4}\right) - \frac{3}{9}$$

$$= \frac{2}{3}$$



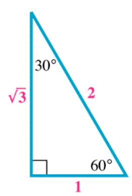
Remember the mnemonic sohcahtoa and mnemonic cast.

Example of Using Coterminal Angles to Find Function Values

- Evaluate each function by first expressing the function in terms of an angle between 0° and 360°.
- cos 780°
  - cos 780° = cos (780° – 2(360°))

    = cos 60°

    $= \frac{1}{2}$



Function Values Using a Calculator

- Calculators are capable of finding trigonometric function values.
- When evaluating trigonometric functions of angles given in degrees, remember that the calculator must be set in *degree mode*.
- Remember that most calculator values of trigonometric functions are *approximations*.

Example

- a) $\sin 38^\circ 24'$
- Convert 38° $24'$ to decimal degrees.
- $38^\circ 24' = 38\frac{24}{60}^\circ = 38.4^\circ$

$\sin 38^\circ 24 = \sin 38.4^\circ$

$\approx .6211477$

- b) cot 68.4832 °
- Use the identity $\cot\theta = \frac{1}{\tan\theta}$.
- cot 68.4832 °

$\approx$ .3942492

Angle Measures Using a Calculator

- Graphing calculators have three *inverse functions*.
- If *x* is an appropriate number, then $\sin^{-1} x, \cos^{-1} x,$ or $\tan^{-1} x$ gives the measure of an angle whose sine, cosine, or tangent is *x*.

Note: Please go over page 15 of your Graphing Calculator Manual.

Example

- Use a calculator to find an angle θ in the interval $[0^\circ, 90^\circ]$ that satisfies each condition.
- $\sin\theta \approx .8535508$

Using the degree mode and the inverse sine function, we find that an angle θ having sine value .8535508 is 58.6°.
We write the result as $\sin^{-1} .8535508 \approx 58.6^\circ$

Example (cont.)

- $\sec\theta \approx 2.486879$

Use the identity $\cos\theta = \frac{1}{\sec\theta}$. Find the reciprocal of 2.48679 to get $\cos\theta \approx .4021104.$

Now find θ using the inverse cosine function.
The result is $\theta \approx 66.289824^\circ$

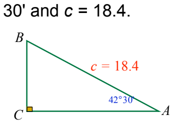
Significant Digits

- A significant digit is a digit obtained by actual measurement.
- Your answer is no more accurate then the least accurate number in your calculation.

Number of Significant Digits	Angle Measure to Nearest:
2	Degree
3	Ten minutes, or nearest tenth of a degree
4	Minute, or nearest hundredth of a degree
5	Tenth of a minute, or nearest thousandth of a degree

How to Solve a Right Triangle Given an Angle and a Side?

- Solve right triangle *ABC*, if *A* = 42° 30' and *c* = 18.4.
- *B* = 90 – 42° 30'
  B = 47° 30'



$$\sin A = \frac{a}{c}$$

$$\sin 42^\circ 30' = \frac{a}{18.4}$$

$$a = 18.4 \sin 42^\circ 30'$$

$$a = 18.4(.675590207)$$

$$a \approx 12.43$$

$$\cos A = \frac{b}{c}$$

$$\cos 42^\circ 30' = \frac{b}{18.4}$$

$$b = 18.4 \cos 42^\circ 30'$$

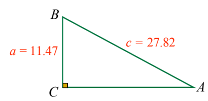
$$b \approx 13.57$$

How to Solve a Right Triangle Given Two Sides?

- Solve right triangle *ABC* if *a* = 11.47 cm and *c* = 27.82 cm.

$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{11.47}{27.82} \approx .412293314$$

$$\sin^{-1} A = 24.35^{\circ}$$

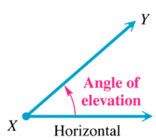


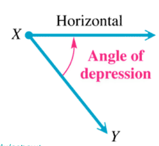
- *B* = 90 – 24.35°
  B = 65.65°

$$b^2 = c^2 - a^2$$
$$b^2 = 27.82^2 - 11.47^2$$
$$b \approx 25.35$$

What is the Difference Between Angle of Elevation and Angle of Depression?

- Angle of Elevation: from point *X* to point *Y* (above *X*) is the acute angle formed by ray *XY* and a horizontal ray with endpoint *X*.
- Angle of Depression: from point *X* to point *Y* (below) is the acute angle formed by ray *XY* and a horizontal ray with endpoint *X*.





How to Solve an Applied Trigonometry Problem?

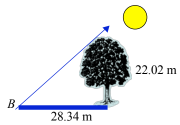
- *Step 1* Draw a sketch, and label it with the given information. Label the quantity to be found with a variable.
- *Step 2* Use the sketch to write an equation relating the given quantities to the variable.
- *Step 3* Solve the equation, and check that your answer makes sense.

Example

- The length of the shadow of a tree 22.02 m tall is 28.34 m. Find the angle of elevation of the sun.
- Draw a sketch.

$$\tan B = \frac{22.02}{28.34}$$

$$B = \tan^{-1}\frac{22.02}{28.34} \approx 37.85^\circ$$



- The angle of elevation of the sun is 37.85°.

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What have we learned?

We have learned to:

1. Express the trigonometric ratios in terms of the sides of the triangle given a right triangle.
2. Apply right triangle trigonometry to find function values of an acute angle.
3. Solve equations using the cofunction identities.
4. Find trigonometric function values of special angles.
5. Find reference angles.
6. Find trigonometric function values of non-acute angles using reference angles.
7. Evaluate an expression with function values of special angles.

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What have we learned? (Cont.)

8. Use coterminal angles to find function values .
9. Find angle measures given an interval and a function value.
10. Find function values with a calculator.
11. Use inverse trigonometric functions to find angles.
12. Solve a right triangle given an angle and a side.
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Credit

Some of these slides have been adapted/modified in part/whole from the slides of the following textbook:

- Margaret L. Lial, John Hornsby, David I. Schneider, Trigonometry, 8th Edition